

# **AT** MICROFICHE REFERENCE LIBRARY

A project of Volunteers in Asia

## Gear Wheels and Gear Cutting

by: Alfred W. Marshall

Published by:

**Model and Allied Publications**  
Argus Books Limited  
P.O. Box 35, Wolsey House  
Wolsey Road, **Hemel** Hempstead  
Hertfordshire HP2 4SS England

Paper copies are \$ 3.00.

Available from:

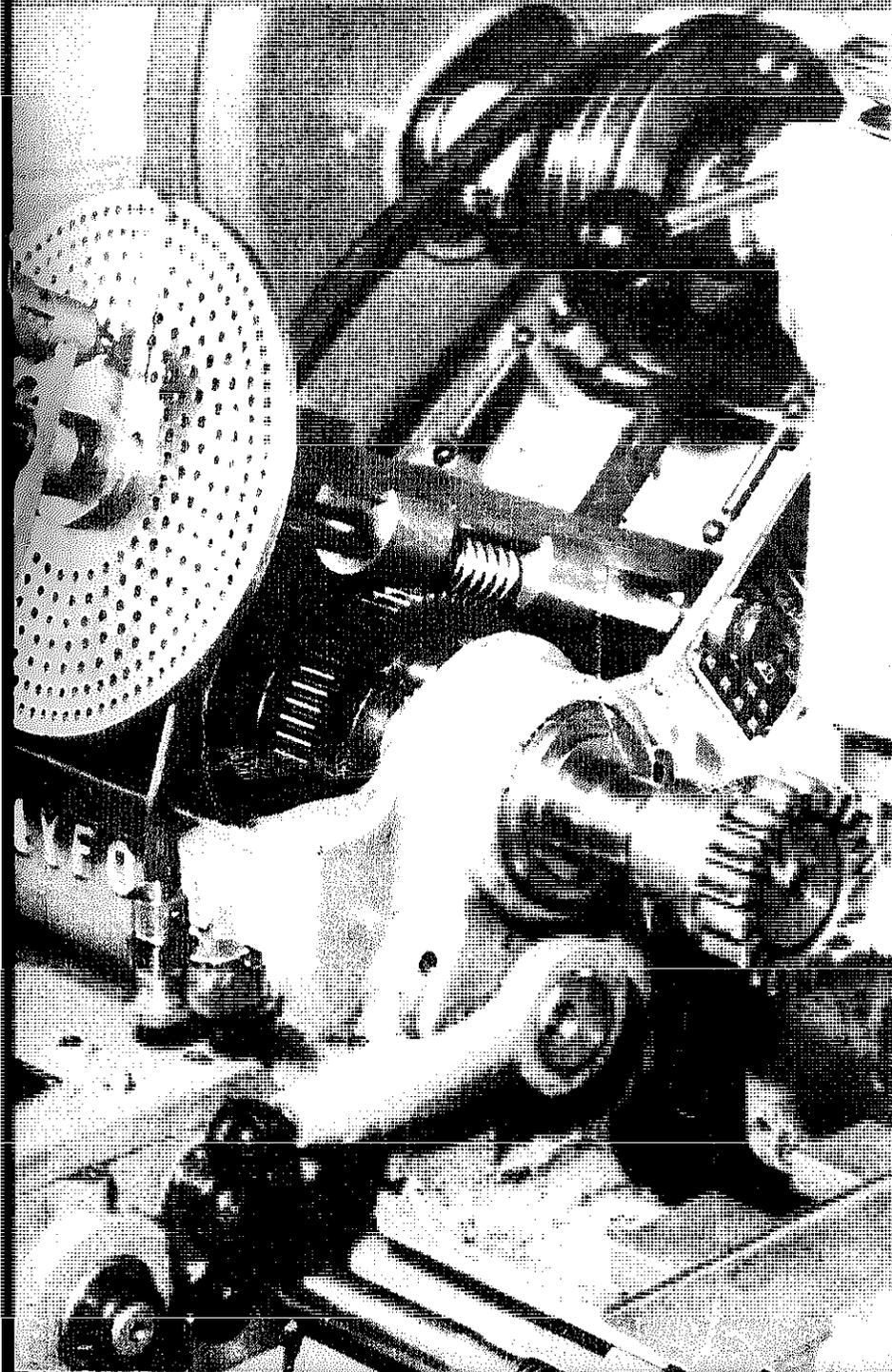
**META** Publications  
P.O. Box 128  
Marblemount, WA 98267 USA

Reproduced by permission of Model and Allied  
Publications.

Reproduction of this microfiche document in any  
form is subject to the same restrictions as those  
of the original document.

# GEAR WHEELS and GEAR CUTTING

BY ALFRED W. MARSHALL



TECHNICAL

MAD

PUBLICATION

# GEAR WHEELS AND GEAR CUTTING

*An elementary handbook on the principles  
and methods of production of toothed gearing*

*By*

**ALFRED W. MARSHALL**

M.I.MECH.E.

*With a chapter on "Cutting Spur Gears in the Lathe"  
by Ian Bradley*

**MODEL & ALLIED PUBLICATIONS**  
**ARGUS BOOKS LTD, ST JAMES ROAD, WATFORD, HERTS**

**Argus Books Ltd.  
14 St James Road, Watford  
Hertfordshire, England**

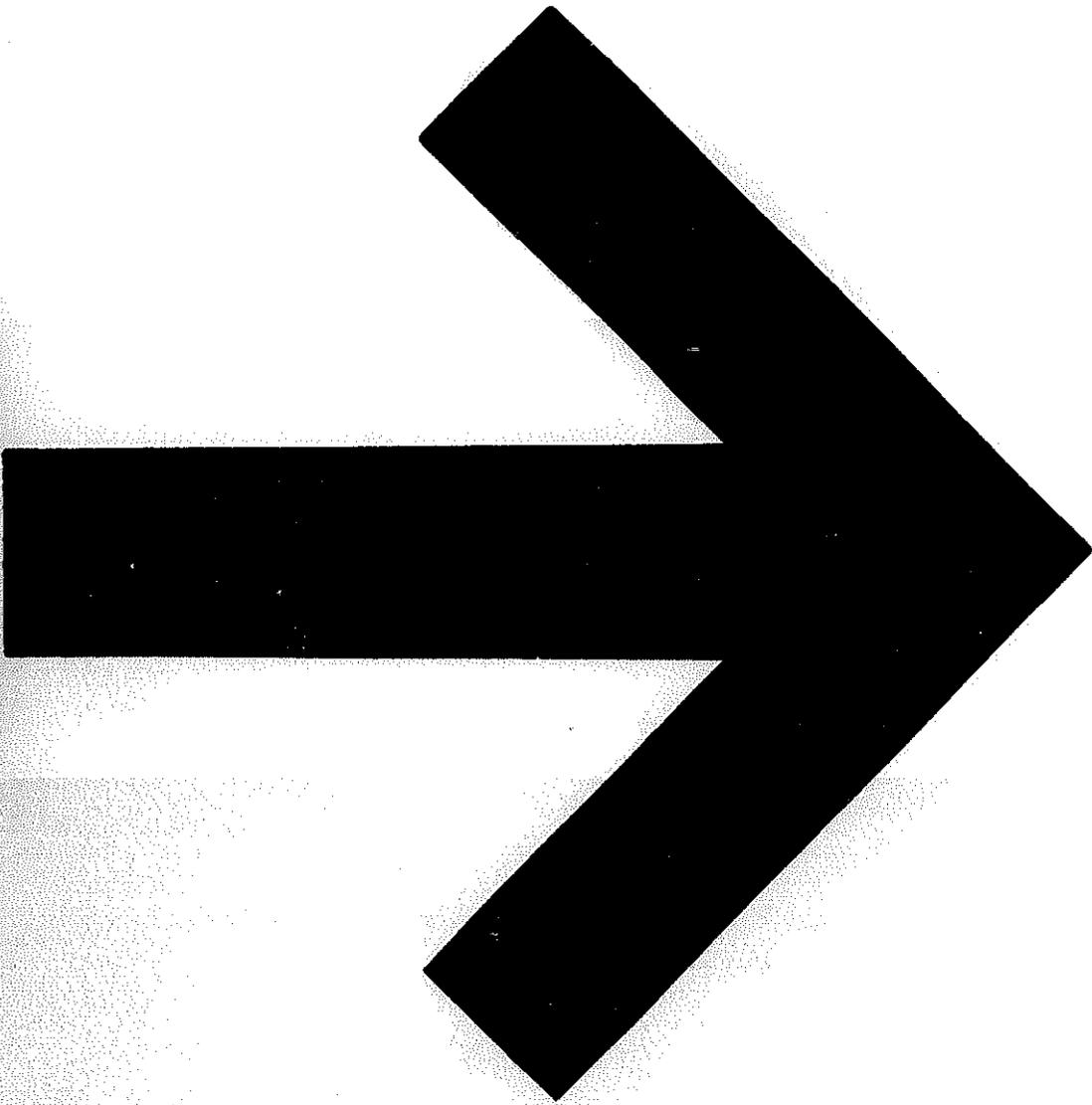
**Second Edition 1947  
Third Edition 1951  
Second impression 1960  
Third impression 1968  
Fourth impression 1971  
Fifth Impression 1973  
Sixth impression 1975  
Fourth Edition 1977**

**ISBN 0 85242 532 5**

**©1977 Argus Books Ltd.**

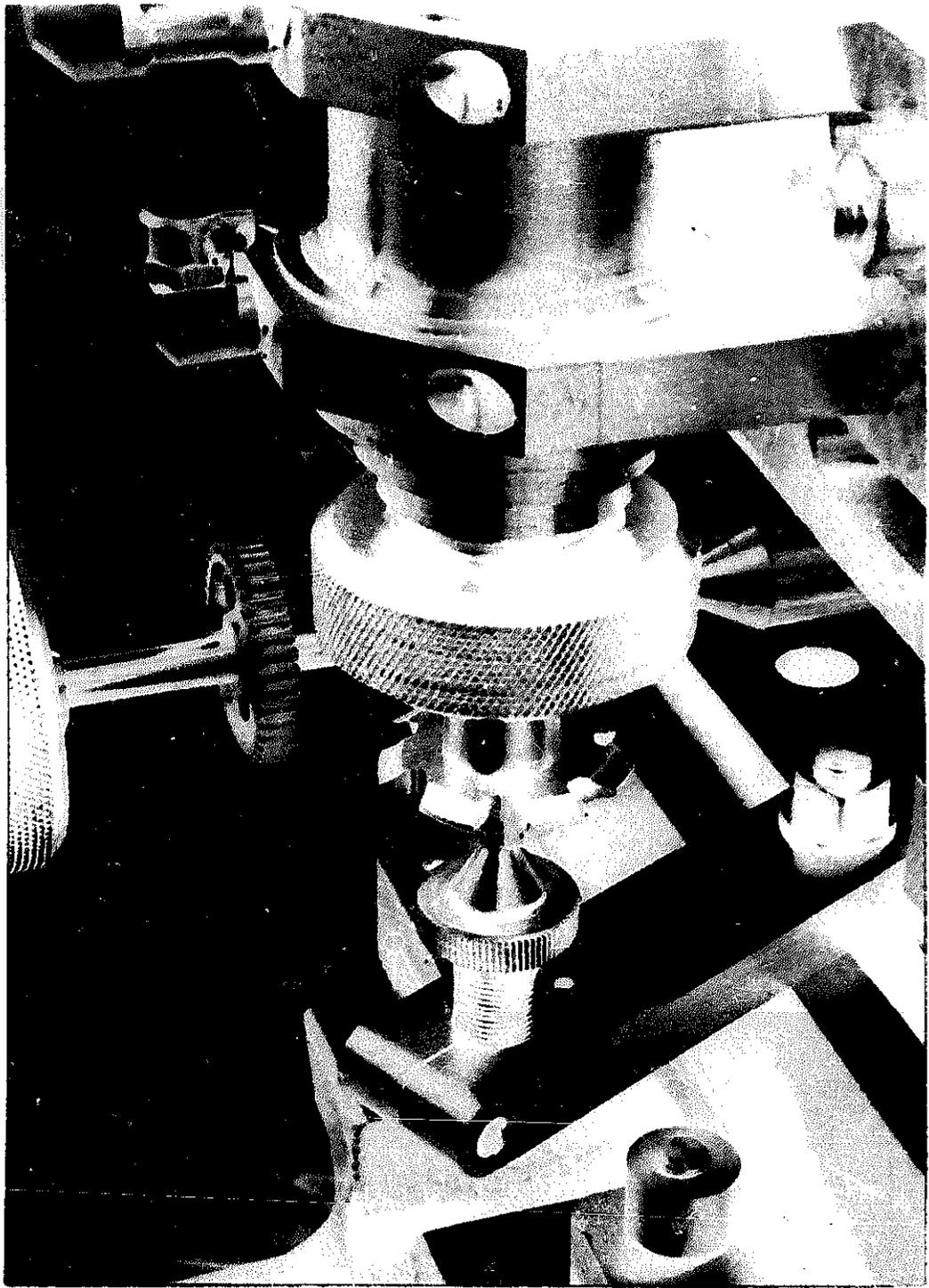
**All rights reserved. No part of this  
publication may be reproduced in  
any form without the prior  
permission of the publisher.**

**Printed Offset Litho in Great Britain  
by Cox & Wyman Ltd, London, Fakenham and Reading**



## CONTENTS

	PAGE
Frontispiece . . . . .	iv
CHAPTER	
I First Principles Explained .. ..	I
II Proportions and Form of Teeth of Gear Wheels	8
III Lantern or Pin Wheels and Racks .. ..	16
IV Bevel Gears . . . . .	25
V Worm Gears . . . . .	33
VI Helical Gears—Spiral Gears—Chain Gearing— Epicyclic Gears—Special Forms of Gear Wheels . . . . .	45
VII Definitions and Calculations for Gear Cutting	57
VIII Gear Cutting in Milling Machines .. ..	64
IX Cutting Spur Gears in the Lathe . . . .	71
X Gear Generating Methods .. .. .	85



Frontispiece.-Cutting the teeth of a small spur gear in the lathe

## CHAPTER I

### First Principles Explained

ONE of the common methods of transmitting motion is by means of wheels which make contact or gear, as it is called, with one another. Movement being given to the first wheel is communicated by it to the second wheel. Any number of such wheels can be geared together—the movement of the first wheel can be communicated to the second wheel, and by the second to the third, and so on. Two such wheels are called a pair; if there are more than two, the arrangement is called a train of wheels. If the edges or surfaces by which contact is made between one wheel and another are smooth, the power is transmitted by means of the friction existing between the surfaces. The wheel which is transmitting the power is called the driving wheel, and the one receiving it is called the driven or following wheel, or just the follower. The wheels may be of equal size or one may be larger than the

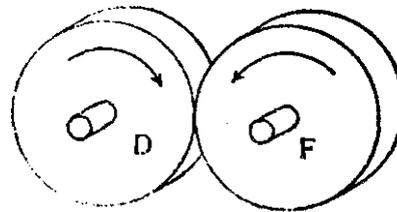


Fig. 1-Diagram showing pair of gear wheels and their relative motion

other. In this latter instance the smaller wheel is called a pinion. Fig. 1 shows a diagram of a pair of wheels in gear. If D is the driver, F is the driven wheel or follower. Fig. 2 shows a diagram of a train of wheels; if A is the driver, its motion will be transmitted by B and C in turn to D. Any one of the wheels can be made the driver; for example, B, which will then communicate

its movement to A, and to D through C. Fig. 3 shows a wheel W and pinion P. The driven wheel will resist the action of the driving wheel. It will do this because some friction must exist

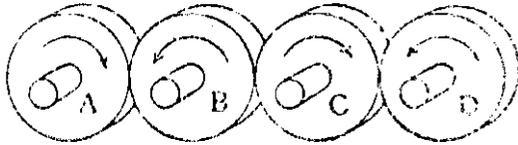


Fig. 2.—Train of gear wheels and relative motion of each wheel

at its bearings, even if no other load is placed upon it. The amount of power transmitted by the driver will vary according to the resistance to motion offered by the driven wheel. If

this resistance is too great to be overcome by the frictional grip existing between the contact surfaces of the wheels, the driven wheel will lose movement and there will be slip between the contact surfaces. To prevent slip the surfaces are cut into teeth, and made to engage positively with each other. By this means an accurate transmission of the motion is ensured. Such wheels are called spur or gear wheels.

Imagine a pair of gear wheels, A and B (Fig. 4) ; B is the driver giving motion to A. If we fix a tooth T upon B to prevent slip, we must cut a groove G in A for it to engage with, or the wheels cannot continue to rotate. A series of such teeth, spaced at equal distances, may be fixed upon the circumference of B, and a series of grooves to receive them cut in the circumference of A. Slip cannot then take place. B is geared into A and drives that wheel positively, or A may be the driver and give motion to B. This positive engagement between the two wheels is entirely due to the teeth T projecting beyond the circumferential surface of B. Matters will be equalised, and the time during which any particular tooth of one wheel is engaged with the other wheel will be prolonged, if teeth are placed upon the circumferences of both wheels. In this instance we should place teeth V upon wheel A for this purpose. We must then cut grooves W in B to receive these teeth. As A is already

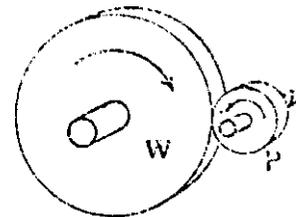


Fig. 3.—Gear wheel and pinion, and their relative motion

cut with a series of grooves, and B is provided with a series of teeth, the new grooves and teeth must be placed at the unoccupied parts of the respective circumferences. The teeth will therefore be placed on the parts T of A (Fig. 5), and the grooves cut in the parts G of B, as indicated by the dotted lines.

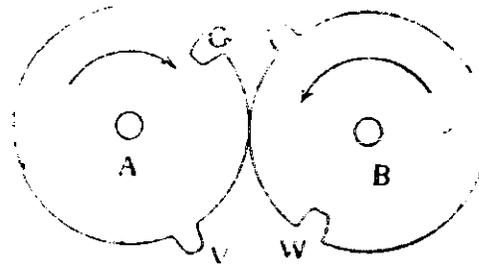


Fig. 4.—Showing action of tooth and space

**The Pitch Circle**

The teeth of a gear wheel are, therefore, made up of two parts, one of which is inside and the other outside the true circumference of the wheel, as indicated by the shaded parts T (Fig. 6.) When designing a pair or train of toothed wheels, we should therefore first imagine them to be without teeth and merely rolling against one another with frictional contact only. In fact, we should plan them as friction gearing and merely add the teeth to the plain wheels thus designed. The circumference of such a plain wheel is called the pitch surface, usually referred to as the pitch circle because, when setting out the gear upon paper, circles are first drawn to represent these pitch surfaces.

The pitch circles shown in Fig. 6 represent the contact surfaces between a pair of plain wheels. The

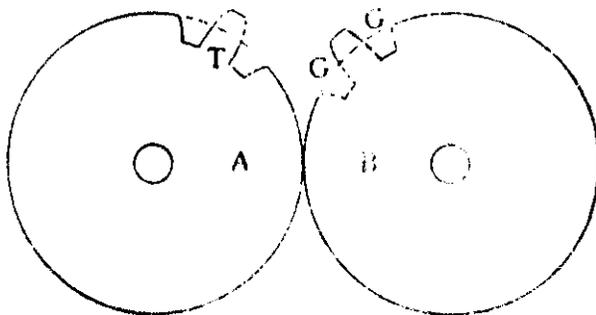


Fig. 5.—Explaining the parts of a tooth

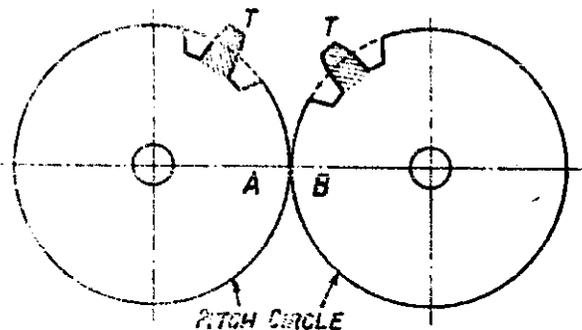


Fig. 6.—Principle of pitch circles

part of the contact surface of the tooth which is outside the pitch circle is called the face, and that part inside the pitch circle is called the flank. The entire portion of a tooth which is outside the pitch circle is called the addendum, and that inside it is the dedendum. When planning a pair or train of wheels, the first consideration is the value or ratio of the gearing. This means the relation between the number of complete revolutions made by the first and last wheels respectively in any given interval of time; or time can be left out of consideration and the value of the gearing be regarded as the number of complete revolutions which the last wheel will make whilst the first wheel makes one complete revolution. The first wheel is considered to be the one which sets the whole train in motion. If the last wheel makes one complete revolution whilst the first wheel also makes one revolution the train is said to be of equal gear ratio. But if we arrange the sizes of the wheels in suitable proportion, the last wheel can be made to give more or less than one revolution for each revolution of the first wheel. If the last wheel has rotated more than once when the first wheel has made one complete revolution, the train is said to be geared up; if less, it is said to be geared down.

### Gear Ratios

The ratio of revolutions is determined by the diameters of

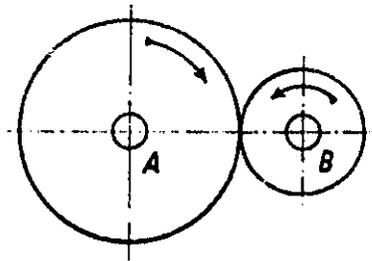


Fig. 7.—Pitch circles to increase revolutions

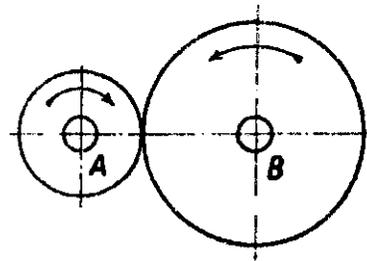


Fig. 8.—Pitch circles to decrease revolutions

the pitch circles. Thus, if the wheels A and B (Fig. 6) are to make equal revolutions, B making a complete revolution for each complete revolution of A, the pitch circles must be equal

in diameter. If B is to make two revolutions for each one made by A, the pitch circle of B must be exactly one-half the diameter of the pitch circle of A (Fig. 7). Suppose that A (Fig. 8) is to make two revolutions for one revolution of B, the pitch circle of B must be twice as large as that of A. Thus the required ratio of revolutions between the driver and driven wheel is determined not by their diameters, as measured over the points of the teeth, but by temporarily leaving the teeth out of consideration and calculating the sizes of the pitch circles alone.

Having decided the diameters of the pitch circles, the diameters of the wheels, measured over the tops of the teeth, are determined by adding an allowance

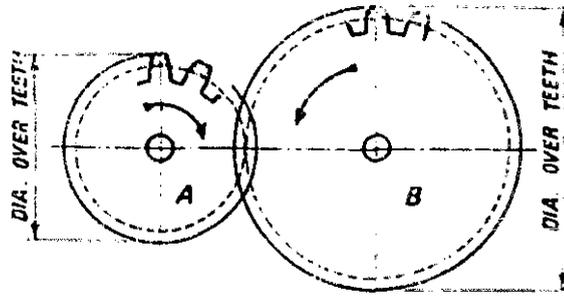


Fig. 9.—Determination of overall diameter of wheels

equal to that part of the teeth which projects beyond the pitch circles. This is shown by Fig. 9, the pitch circles being the dotted lines and the full circles the over-all diameters of the wheels. The part of the teeth which projects beyond the pitch circle is shaded. Patterns or blanks from which the wheels will be made would, therefore, be turned to this over-all diameter which thus provides the requisite allowance to complete the teeth. When machining the wheels in the lathe, it is often the practice to mark a line representing the pitch circle upon the side of the wheel. This serves as a guide when cutting the teeth and also for meshing the wheels correctly.

### The Tooth Pitch

The ratio of revolutions between one wheel and another also depends upon the relative number of teeth. If wheel A has 20 teeth and wheel B 30 teeth, A will rotate one and a half turns to one complete revolution of B. Therefore we must not only design the pitch circles so that their diameters are

of the correct ratio, but we must also make the numbers of the teeth to correspond. To some extent this question decides itself, because the teeth upon A must be spaced at a distance apart to correspond with the spacing of the teeth upon B, or the two sets will not fit properly together ; the numbers of teeth should, however, always be calculated and made to correspond with the diameters of the pitch circles. The distance from the centre of one tooth to the centre of the next is called the pitch and is measured along the pitch circle. If the two wheels are to gear properly together, the pitch of the teeth upon A must be of the same pitch as those upon B. When determining the number of teeth for, say, wheel B, it may be found that any number which gives a reasonable pitch and is a convenient fraction of an inch, such as  $\frac{1}{8}$  in. or  $\frac{1}{4}$  in., will not divide the pitch circle of A into the correct number of teeth. If the wheel centres are not fixed, the matter may perhaps be adjusted by a slight alteration in the sizes of the pitch circles, still keeping them to the desired proportion. If the wheel centres cannot be altered, the pitch of the teeth will have to be adjusted accordingly. There is another method of reckoning the pitch. Instead of measuring it along the circumference, it is measured as so many teeth per inch diameter of the pitch circle. Thus, if a wheel having a pitch circle diameter of 3 in. is to have 24 teeth, they are said to be of 8 diametral pitch, because there are 8 teeth in 1 in. of the pitch circle diameter. Awkward fractions of an inch can thus be dealt with in a simple way ; No. 8 diametral pitch would be .393 circumferential pitch. If the circumference of the pitch circle is made of such a size that fractions are avoided, the diameter may be some awkward dimension. By working to diametral pitch, the pitch circle diameter can be easily measured and set at. Tool makers generally use this method and supply a variety of cutters made to diametral pitch. Therefore, as a rule, there is no difficulty in planning the gear teeth and obtaining the corresponding gear cutters.

**Rules for Calculating Gear Wheels**

The following formulae are useful for calculating gear wheels:—

**THE CIRCULAR PITCH MULTIPLIED BY THE NUMBER OF TEETH AND DIVIDED BY 3.1416 WILL GIVE THE DIAMETER OF THE PITCH CIRCLE.**

**THE DIAMETER OF THE PITCH CIRCLE MULTIPLIED BY 3.1416 AND DIVIDED BY THE NUMBER OF TEETH WILL GIVE THE CIRCULAR PITCH.**

**THE DIAMETER OF THE PITCH CIRCLE MULTIPLIED BY 3.1416 AND DIVIDED BY THE PITCH WILL GIVE THE NUMBER OF TEETH.**

**TO OBTAIN THE DIAMETRAL PITCH FROM THE CIRCULAR PITCH, DIVIDE 3.1416 BY THE CIRCULAR PITCH.**

**TO OBTAIN THE CIRCULAR PITCH FROM THE DIAMETRAL PITCH, DIVIDE 3.1416 BY THE DIAMETRAL PITCH.**

The quantity 3.1416 ( $\frac{22}{7}$  to four decimal places) is the ratio between the circumference and diameter of any circle, named and denoted by the Greek letter pi ( $\pi$ ).

These and other formulae are shown in greater detail, and with examples, in Chapter VII.

## CHAPTER II

### Proportions and Form of Teeth

THE size of gear teeth is determined according to the power they have to transmit. They tend to break at the lowest portion—that is, at the root. If the wheels are well fitted, and the teeth make contact throughout the entire width, they will resist the stress much better than if they are inaccurately fitted. In the latter case, they may make contact at some place near the edge so that the stress is concentrated mainly upon a small portion of the metal. The power which a gear wheel can transmit increases with the speed of the rotation. If a wheel has a slow speed of rotation, to transmit a given amount of power its teeth must be larger than they need be if the speed were higher. Generally, there will be two or more pairs of teeth in contact simultaneously, so that the pressure is distributed over two teeth. All the small gearing likely to be used will probably have sufficient strength when the teeth are made of standard proportions, so that no calculations need be made for this. If the teeth are to be cut from the solid—and this is the best method for small wheels—the blanks can be given to a gear-cutter who will select a suitable tool, and it will only be necessary to state the sizes of the pitch circles and the number of teeth to be cut in each wheel. When deciding upon the number of teeth, arrange to have as many teeth as possible consistent with strength and wear. It is not advisable to have less than seven teeth in any wheel.

#### Proportions of Teeth

The teeth are usually proportioned according to the length

of the pitch, but different makers vary the dimensions to a small extent. The well-known authority, Professor Unwin, in his *Elements of Machine Design*, gave the proportions shown in Fig. 10, the unit being the pitch. These dimensions show that the thickness of a tooth measured on the pitch circle should be slightly less than the width of the interval between the teeth (called the space). Also that there will be a clearance space between the point of the tooth of one wheel and the bottom of the space of the wheel into which it is geared. The width of the wheel is usually about 2 to  $2\frac{1}{2}$  times the pitch. When a pair of wheels is in gear, the pitch circles should touch. If such proportions for the teeth and spaces as are given in Fig. 10 have been adopted, there will then be a small amount of play between the teeth, as the thickness of the teeth is slightly less than the width of the spaces, and the height above the pitch circle is less than the depth below. This clearance allows for very small irregularities, and enables the wheels to run without jamming; it should not be gained by extending the distance between the centres of the wheels.

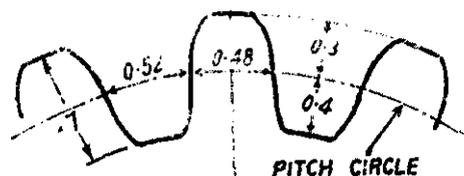


Fig. 10.—Tooth proportions

### Mortise Wheels

There is a kind of gearing used in mill work called mortise wheels. Here, one wheel of a pair is fitted with wooden teeth dovetailed into slots in the rim. When these wheels are used the proportions of the teeth are altered, the wooden teeth being made thicker than the metal teeth of the wheel with which it is geared. The object of the gearing is to reduce noise, and it would hardly be used for small gearing except as a model of a large gear.

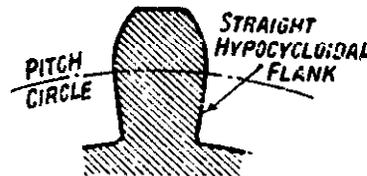
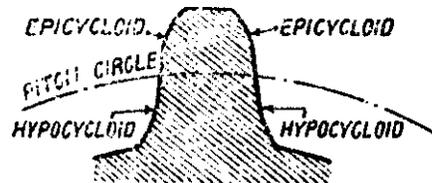
### Cycloidal Teeth

The teeth of gear wheels have to be made to an exact shape, and it is not sufficient to make them of any pattern that will

allow of engagement and disengagement during rotation without binding. There is a further consideration : the teeth must be of such a shape that the relative velocities of the pitch circles will not be disturbed as they roll one against the other. The pitch circles should continue to roll as if there were no teeth and no slip. Mathematicians have discovered that if the teeth are shaped according to certain well-known geometrical curves, this condition will be fulfilled. The three curves which are used in practice are the epicycloid, the hypocycloid, and the involute. When made on the cycloidal principle the contact surfaces of each tooth are composed of two curves. That part which is outside the pitch circle is an epicycloid curve, and that part which is inside the pitch circle is a hypocycloid curve (see Fig. 1 1) . A cycloid is the curve which is described by a point fixed at the circumference of a circle when that circle is rolled in contact with a straight line. An epicycloid is the curve which would be described by the point if the circle were rolled upon the circumference of another circle. A hypocycloid is the curve which would be described if the circle were rolled in contact with, but inside, the circumference of another circle. The rolling circle is termed the generating circle.

If the curves of the faces of the teeth on one wheel are formed by the same rolling circle as that used to form the flanks of the teeth on the wheel with which it is to gear, the relative velocities of the pitch circles will not be disturbed by the engagement of the respective teeth. This is actually done in practice and the curves are then produced by rolling a disc, representing the curve-generating circle, upon another disc or template representing the pitch circle, or these curves are drawn with compasses to some geometrical construction which gives arcs of circles very closely approximating to the real curve. Methods of this kind are given in text-books on gearing and machine construction and in engineering pocket-books. The same generating circle can be used to describe the curves for the faces and flanks of the teeth of each wheel ; this is convenient and usual in practice, though two generating circles could be used—

one for the flanks of the driver teeth and faces of the driven teeth, and the other for the faces of the driver and flanks of the driven teeth. If more than two wheels are in gear together, or if a number of wheels are required to gear indiscriminately with one other—as in the case of a set of change-wheels for a lathe or other machine—it is necessary to use one circle only to generate the curves for the faces and flanks of the teeth of all the wheels.



Figs. 11 and 12.—Tooth forms

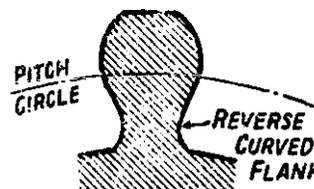


Fig. 13—Weak tooth form

### Teeth with Straight Flanks

Many wheels are made with teeth which have straight radial instead of curved flanks. This is quite correct, because a hypocycloid generated by a circle whose diameter is equal to the radius of the pitch circle inside which it rolls is a straight line (Fig. 12). The generating circle should not be made larger than

this, as the straight line then becomes a reverse curve, producing a weak form of tooth at the root, as indicated by Fig. 13. For this reason the diameter of the generating circle to form the teeth of a set of wheels of different sizes is usually made equal to the radius of the pitch circle of the smallest wheel. The flanks of the teeth of that wheel will then be straight lines, and those of all the others will be curves. But all will be hypocycloids, and the teeth will not be weak at the root. According to Molesworth, the best diameter of the generating circle is given by 2.22 times the pitch, provided the number of teeth in any one of the wheels is not less than fourteen. If the number be less, the diameter of the generating circle should be equal to the number of teeth multiplied by the pitch and divided by 6.3.

### Involute Teeth

Teeth shaped upon the cycloidal principle preserve the relative velocities of the pitch circles only if the wheel centres are at the proper distance apart. If the centres are further apart so that the pitch circles do not rotate in contact, the relative velocity is not maintained. There will, however, still be the ratio of revolutions between the wheels—obviously this cannot alter; but the wheels themselves will not revolve with an even motion. Each will be intermittently going faster and slower as the teeth come into and pass out of engagement. This will tend to set up vibration and noise and may be a disturbing factor in delicate mechanisms. If the teeth are shaped on the involute principle, the relative velocities of the wheels will not be affected if the centres are spread apart or one brought more closely together, provided the change of position is small. Any normal wear of the bearings would thus not interfere with the proper action of the teeth. The involute is a geometrical curve produced by the end of a stretched cord which is being unwound off a cylinder or the circumference of a circle; it would also be produced by the end of a straight line which is being rocked upon the circumference of a circle. In either case the circle is called the base circle, and the curve produced is called the involute of that circle. Teeth shaped upon this principle do not have two curves for their contact surfaces; the face and flank at each side of a tooth is formed by one and the same involute curve (see Fig. 14). The curve is produced by the end of the line B, which represents the cord or straight line rocking upon the base circle. Such teeth are of strong shape, and all wheels with involute teeth will work correctly together if the teeth are of the same pitch and the same angular line of contact. That is a line L C (Fig. 14) making contact with the base circle and passing through the pitch point P. For any pair of a set of wheels to work together, the radii of the base circles must bear the same proportion as the radii of the pitch circles. A curve consisting of an arc of a circle can be produced which is very near to the true involute curve; geometrical methods for this

are given in text-books on gearing and machine construction and in engineering pocket-books. According to some authorities, teeth shaped to the involute curve exert a thrust along the line joining the centres of the wheels (called the line of centres), thus exerting extra pressure upon the bearings ; therefore the involute curve should not be adopted if the wheels have to transmit a considerable amount of force. Others do not agree with this. Hawkins, in his book on the teeth of wheels (largely translated from the work of the French mathematician Camus), describes an experiment made by him with a pair of wheels having involute teeth. The wheels were tried with the teeth engaging at various depths, and did not show any tendency to thrust the centres apart until they were placed with the teeth only engaged to a depth of  $\frac{1}{4}$  in. out of a total depth of  $1\frac{1}{2}$  in. ; even then the tendency to separate the centres was very slight. The involute form of teeth is favoured in modern practice, and is now very generally used. If the angle A (Fig. 14) be made as large as practicable, involute teeth appear to give good results in working. The effect of increasing the angle A is to bring the circumference of the base circle close to that of the pitch circle, and so shorten the teeth.

### Internal Toothed Gears

Gear wheels can be made in the form of a ring with teeth inside the circumference instead of outside. These wheels are called internal or annular gear wheels. Such a wheel can obviously only gear with another which has external teeth and is smaller in diameter, because the second wheel is placed within the first. These wheels are calculated and set out according to the diameters of their pitch circles, as in the case of external toothed gears. The pitch circle of the pinion P is inside that of the wheel W (Fig. 15) and touching at the pitch point.

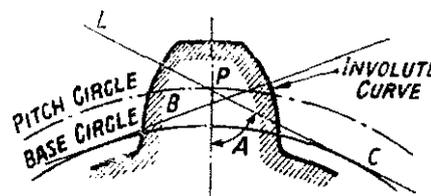


Fig. 14.—Involute tooth form

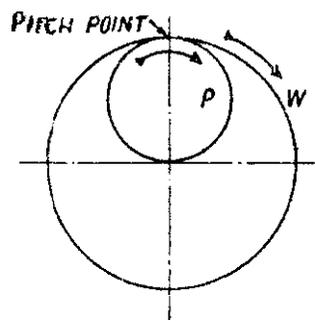
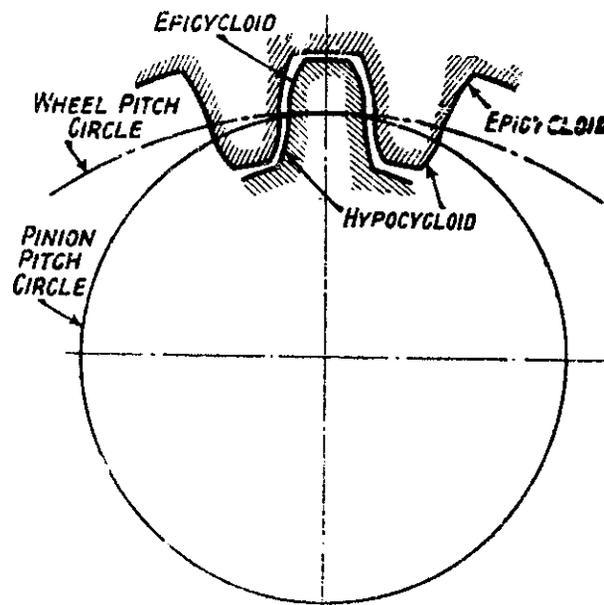


Fig. 15.—Pitch circle principle applied to internal gear

If the diameter of the pitch circle of the wheel *W* is four times that of the pinion *P*, the latter will make four revolutions for one revolution of the wheel, and so on. The numbering and proportions of the teeth apply as in the case of external gears. Cycloidal or involute curves can be used for the shape of the teeth. If cycloidal curves are used, they will be transposed in the case of the teeth of the wheel. The curve-generating circle for the faces of the teeth will roll inside the pitch circle, and therefore produce a hypocycloid, and that producing the flanks will roll outside the pitch circle and therefore produce an epicycloid (see Fig. 16). An internal gear wheel can engage with several pinions simultaneously placed at different positions along its pitch circle; the pinions may be of various sizes. In such an instance the curve-generating circle for all the teeth should have a diameter equal to half that of the

Fig. 16.—Geometric curves of internal gear teeth

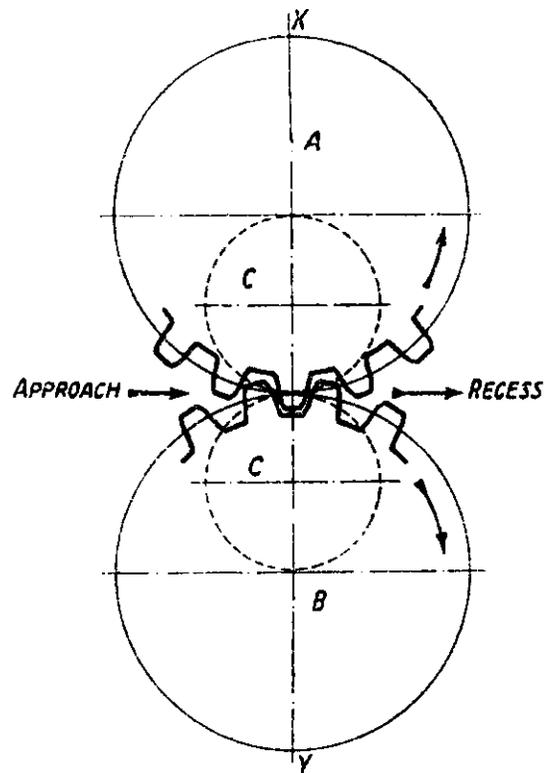


pitch circle of the smallest pinion. There is an important difference between the working of an internal and external pair of toothed wheels. If the wheels are external, they rotate in opposite directions, but if internal, they rotate in the same direction, as indicated by the arrows (Fig. 15).

The Contact of Teeth

If the teeth are made as so far explained, with a part projecting beyond the pitch circle as well as a part inside it, any pair will come into contact as they

Fig. 17.—Principle of engagement and disengagement of teeth



approach the line of centres, and this contact will be maintained to some distance after they have passed the line of centres. For example, a pair of wheels A B, Fig. 17) are in gear. The teeth of wheel

A come into contact with those of B to the left of the line of centres X, Y, as the wheels rotate in the direction indicated by the arrows. They are then said to be engaging. After passing the line of centres they are said to be disengaging. The contact is maintained for some distance, but finally ceases as each pair of teeth passes out of gear. The distance through which they make contact when engaging is called the arc or angle of approach, and that through which they move when disengaging is called the arc or angle of recess. The actual path followed by the contact point is a curve in the case of cycloidal teeth, which consists of arcs of the tooth-generating circles C, C. (Fig. 17a, Chap. III), any particular tooth commencing to make contact at a point in one circle, and leaving contact at a point in the other circle, as indicated by the full line which is termed the path of contact, also the arc of action. In the case of involute teeth the path of tooth contact is along a straight line, such as L C (Fig. 14), commencing inside one pitch circle and ceasing inside the other pitch circle.

## CHAPTER III

### Lantern Or Pin Wheels and Racks

THE teeth of gear wheels rub together whilst in motion, causing friction and wear of the surfaces, but these are reduced by designing the teeth so that the path of contact is as short as possible. The friction arising during engagement in the arc of approach—this would be to the left of X, Y (Fig. 17a)—is considered to be more detrimental than that which occurs during engagement in the arc of recess—this would be to the right of X, Y (Fig. 17a). The teeth rub to greater disadvantage

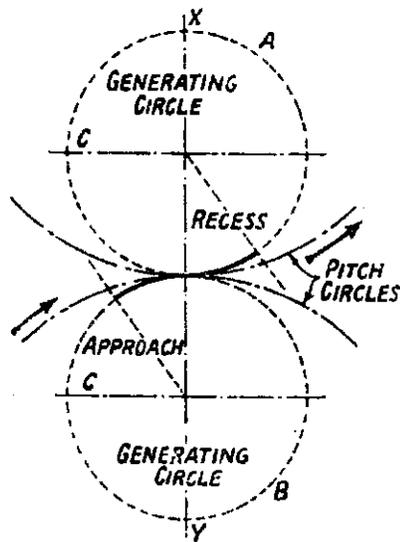


Fig. 17a.—Path of contact of cycloidal teeth

when coming into contact than when disengaging. On this account, designers of wheel gearing, in which it is of special importance that friction and wear should be eliminated as much as possible, such as watch and clock gearing, prefer teeth which only make contact when they have reached the line of centres. Engaging friction is thus reduced to a minimum. This action will be accomplished if the driven-wheel teeth are made without points—that is, they would not project beyond the pitch circle. For example, wheels having complete teeth, such as Fig. 17, come into engagement before the line of centres X, Y. At this stage, the flanks of the teeth on wheel B make contact with the faces of the teeth on wheel A, assuming that B is the driver. After passing the line of centres

the faces of teeth on wheel B make contact with the flanks of the teeth on wheel A. Therefore, if we desire that contact shall only be made after the line of centres, the points of the teeth on A should be removed. It follows from this that the roots of the teeth on B will not be required, and as the engagement is only to take place at, or after, the line of centres, B must be the driver. If A be the driver, the engagement will take place only before the line of centres, which is, here, the condition to be avoided. The teeth having points are therefore put on the driver, and those having no points are on the follower. If cycloidal teeth are used, as they should be, the curve of those upon the driver will be an epicycloid, and that of the teeth upon the driven wheel a hypocycloid. This leads to two distinct forms of teeth for the driven wheel. If the curve-generating circle has a diameter equal to the radius of the pitch circle of the driven wheel, the hypocycloid becomes a straight line (as previously explained), and the teeth have merely straight, radial lines for the contour of their flanks. In the second case, the curve-generating circle is made to a diameter equal to that of the pitch circle of the driven wheel; the hypocycloid then becomes a point and the teeth may be pins projecting at right angles to the side of the wheel. This is the only instance in which the hypocycloid formed by a generating circle of such a proportion to the pitch circle is used for wheel teeth.

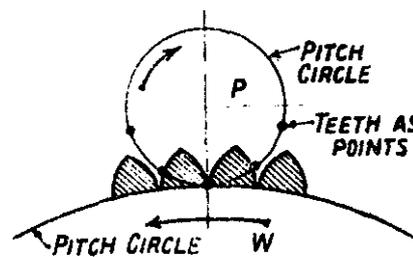


Fig. I&-Principle of wheel and lantern pinion

### Pin Wheels or Lantern Pinions

When the curve-generating circle is made of a diameter equal to that of the pitch circle in which it rolls, and the hypocycloid becomes a point, some practical modification is necessary, because the teeth are, theoretically, merely points, which, according to Euclid's definition, have position but no magnitude.

Obviously the teeth must have some thickness, and in practice they become pins. Fig. 18 is a diagram showing the teeth of a pinion (P) as points, which represent the hypocycloid we have produced by using a curve-generating circle with a diameter equal to that of P. The teeth on the wheel W are entirely outside the pitch circle, and the faces are epicycloids produced by rolling the circle which has generated the point teeth of P upon the pitch circle of W. To make a practical working construction, we fit cylindrical pins to P to form teeth. This is quite justifiable, as a circle is the equivalent of a point, and therefore in the particular instance is logically a hypocycloid. The teeth on W as originally formed to engage with points are represented by dotted lines (Fig. 19). If we enlarge the points so that they become pins, it will be necessary to cut away some portion of the teeth of W to provide room for the pins to engage between the teeth; as already explained, we may not alter the distance between the centres of the wheels. Space for the pins is provided by cutting away a portion equal to half the diameter

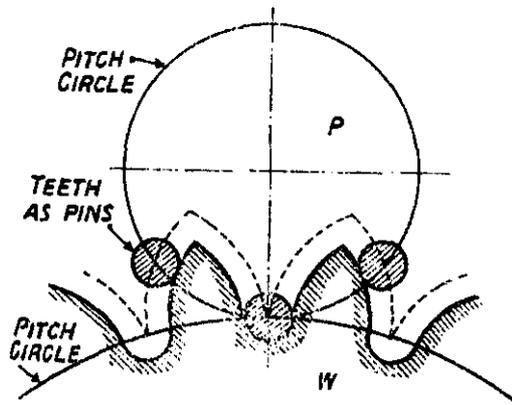


Fig. 19.—Practical gear wheel and lantern pinion

of one of the pins from each face of a tooth along a line parallel to the original curve of the face. This will leave the teeth with the size and shape indicated by the full lines (Fig. 19), the curves still being epicycloids. To complete the clearance space, a semi-circular space is cut away below the pitch line of W between each pair of teeth.

This procedure may be understood by imagining the pin to be a milling cutter moving with its centre coinciding with the original line of the teeth, and thus cutting away the amount of metal necessary to allow the pins to engage. In such a gear, if W is the driver, the engagement of the teeth of the pair of wheels will take place principally after the line of centres. As this is really

the object for which the particular arrangement is designed, whenever it is used, the wheel having the projecting teeth is always made to drive the one provided with pins. If the pins are made to drive the teeth, the engagement takes place before the line of centres, and the object of the design is lost. The

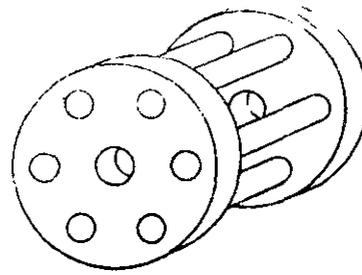


Fig. 20.—Lantern pinion  
or pin wheel

arrangement is much used in clockwork, the wheels with the pins appearing in the familiar form shown in Fig. 20 ; these are called lantern pinions, from the resemblance to a lantern. In the main train of wheels of a clock, the driving force passes through the gearing from the great wheel, which is driven by the spring or weight to the escapement. The wheels, therefore, drive the pinions, and the latter can be provided with pins as teeth, hence the extensive use of lantern pinions for clockwork. Incidentally, these have the advantage of being very strong and durable. The name pin wheel is also applied to this form and the pins are termed rungs, staves, or rounds. Gears of this kind may also be made in the form of a rack and pinion, as will be described later.

### Clock and Watch Gearing

It is also possible to design a pinion with teeth having radial flanks to engage after the line of centres. The number of teeth must be at least ten, and it may be necessary to cut the spaces between the teeth of the pinion with extra width. There would thus be a certain amount of play between the wheels, but this is permissible in clock-gearing where the teeth move slowly, and are kept in contact by a steady constant pressure. Though the teeth of such pinions need not project beyond the pitch circle, they are usually made with a round end projecting beyond the pitch circle by an amount equal to half the thickness of the tooth, in order to ensure smooth engagement as the teeth come

into action. Clock and watch gearing require large wheels driving very small pinions, and work under special conditions. Anybody contemplating the construction of a time-keeping or similar mechanism should consult a treatise on clock and watch gearing. The principles upon which the teeth of the wheels are constructed are, however, precisely those which govern the design of wheel-gearing for machinery in general. As smooth action is very important, the teeth must be of correct design to preserve the relative velocities of the pitch circles. Practical modifications would be introduced to meet the peculiar conditions of clock gearings. For example, the spaces at the roots of the teeth of *W* (Fig. 19) would probably be cut somewhat deeper than indicated, and made rectangular instead of being semicircular; the sides are then radial lines. The teeth of *W* would be of less width than the spaces between the pins on *P*, this amount of play permitting engagement to take place at or very near to the line of centres. The best length for the teeth of *W* would probably be found by experiment.

### Rack and Pinion Gears

A rack and pinion gear may be considered as a pair of toothed wheels, one of which, the rack, has a pitch circle of infinite radius represented by a straight line. The teeth can therefore be shaped according to the principles already explained. Provided the rack is made of sufficient length, the pinion can be made to give any desired number of revolutions for one stroke of the rack, or the rack made to move for any desired length of stroke for one revolution of the pinion. The gear is planned in accordance with the principle used for designing a pair of toothed wheels. The pinion *P* is represented by its pitch circle, and the rack *R* by a pitch line (Fig. 21), which is really the pitch circle of a second wheel stretched out to form a straight line. The two pitch lines touch one another at the pitch point *X*. The pinion may drive the rack, or, conversely, the rack may drive the pinion. As with a pair of wheels, the teeth should be of such a shape that the relative velocities of the pitch circle and line are

maintained. The length of stroke which the rack will make for one revolution will depend upon the diameter of the pitch circle

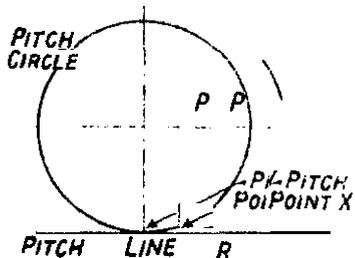


Fig. 21.—Pitch circle principle applied to rack and pinion

of the pinion. When planning a rack and pinion, therefore, the positions of the pitch circle of the pinion and pitch line of the rack should be determined first without regard to the teeth of either. The distance the rack will move for one revolution of the pinion will be equal to the circumference of the pitch circle of the pinion. Thus, if the diameter of the pitch circle of

the pinion is 4 in., the rack will move nearly  $12\frac{1}{2}$  in. for each revolution of the pinion. Conversely, if the rack drives the pinion, the latter will be rotated one complete revolution if the rack be moved through a stroke of approximately  $12\frac{1}{2}$  in.

### Driver or Follower

If the rack or pinion is to be indiscriminately either driver or follower, the teeth should be partly formed outside the pitch lines and partly inside, as

in the case of a pair of wheels. The curves of the teeth of the pinion will be formed with the faces epicycloid and the flanks hypocycloid. The curves of the teeth of the rack will make the faces cycloid, and the flanks cycloid also, because in each instance the curve-

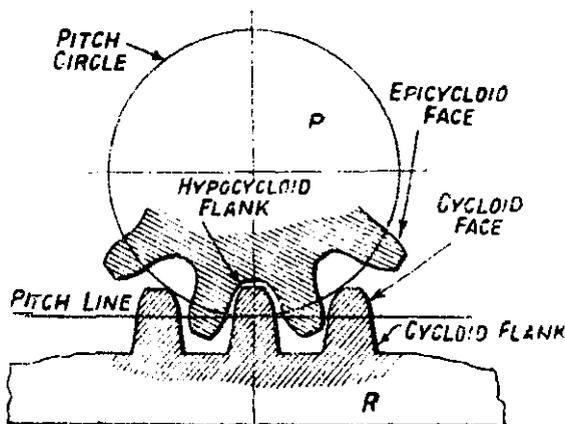


Fig. 22.—Geometric curves of rack and pinion teeth

generating circle is rolled upon a straight line (Fig. 22). The diameter of this generating circle may be anything not exceeding the radius of the pitch circle of the pinion, and the same generating circle can be used to form the whole of the curves. If a

set of wheels is required of different diameters and numbers of teeth, any one to work with the rack, the generating circle should be equal to the radius of the pitch circle of the smallest wheel. The pitch of the teeth is measured on the circumference of the pitch circle of the pinion and along the pitch line of the rack. The pitch may be expressed as diametral pitch in terms of per inch of the pitch circle diameter of the pinion, as previously explained ; it will thus also apply as pitch in the number of teeth per inch length of the rack. When the curve-generating circle has a diameter

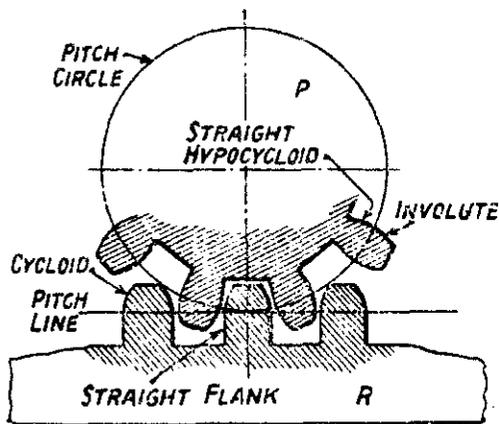


Fig. 23.—Teeth of rack and pinion with straight flanks

equal to the radius of the pinion pitch circle, the teeth of the pinion will have straight radial lines for the flanks, as previously explained, and the rack teeth will have curved lines for both faces and flanks, the curves being a cycloid. The teeth of the rack may, however, be made to have straight radial flanks. As the radius of a straight line is of infinite

length, the flanks of such teeth will be straight lines perpendicular to the pitch line. The faces will be a cycloid formed by the generating circle, which produces the straight radial flanks of the pinion. The faces of the pinion teeth, however, should not be an epicycloid curve, because they should be produced by the generating circle which has produced the so-called radial flanks of the rack. But this circle is one of infinite radius—in fact, it is a straight line. Therefore, the faces of the pinion teeth should be curves produced by rolling a straight line upon the circumference of the pitch circle ; the curve should thus be an involute of the pitch circle of the pinion (see Fig. 23).

#### Pin Teeth for Rack and Pinion Gear

The rack or the pinion may be fitted with teeth in the form of pins on the principle explained with reference to Figs. 18 to

20. As in the case of a wheel and pinion, the pin teeth should be on the follower and not on the driver. If they are placed upon the pinion, the rack should therefore be the driver ; if they are placed upon the rack, the pinion should be the driver. The pins represent a hypocycloid produced by a curve-generating circle having a diameter equal to that of the pitch circle of the wheel upon which they are placed. Therefore, if they are placed upon the pinion, the teeth of the rack will be formed by a cycloid curve produced by a point on the pitch circle of the pinion when it is rolled upon the pitch line of the rack. The rack teeth will be composed of the part which projects above the pitch line, so that they will have faces only and no flanks (see Fig. 24). If the pin teeth are placed upon the rack they represent, as before, a hypocycloid

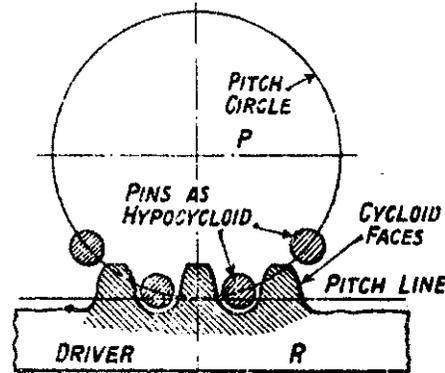


Fig. 24.—Rack and lantern pinion ; tooth shapes

produced by a curve-generating circle will have a diameter equal to the pitch circle upon which the teeth are placed. In this case, the pitch circle is a straight line, therefore the curves of the teeth of the pinion should be produced by rolling a straight line upon its pitch circle. The curve thus produced will be an involute of that pitch circle. The teeth of the pinion are therefore shaped to an involute curve, and consist of the part which projects beyond the pitch circle, no flanks being required (see Fig. 25). Gears on this principle

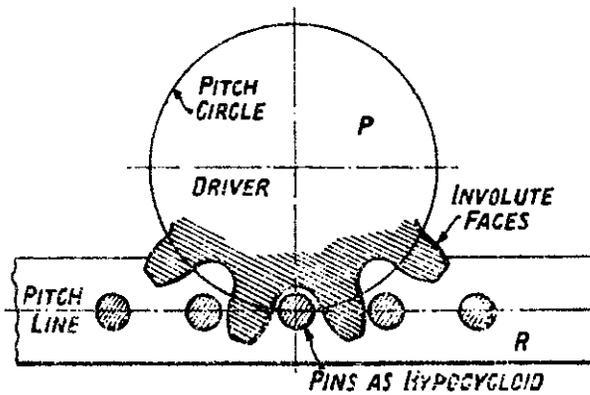


Fig. 25.—Pinion and rack with pin-teeth ; tooth shapes

should work very smoothly if properly made, as the friction between the teeth takes place principally after they have passed the line of centres. Theoretically, the pin teeth are points, as in Fig. 18; the spur teeth of the driver are cut away to allow for the thickness of the pins, as reference to Fig. 19 will explain.

### Involute Teeth for Rack and Pinion Gear

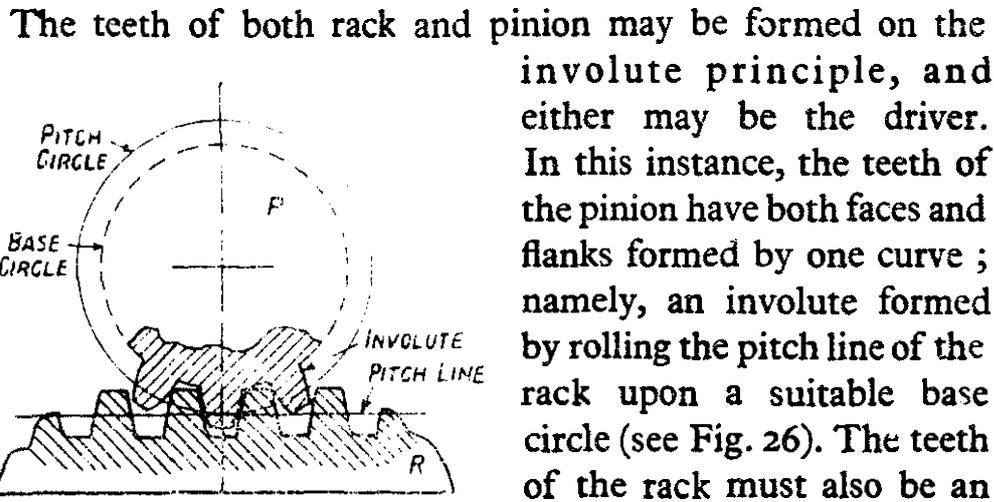


Fig. 26.—Involute form of rack and pinion teeth

The teeth of both rack and pinion may be formed on the involute principle, and either may be the driver. In this instance, the teeth of the pinion have both faces and flanks formed by one curve; namely, an involute formed by rolling the pitch line of the rack upon a suitable base circle (see Fig. 26). The teeth of the rack must also be an involute in theory. But the involute of a circle of infinite radius is a straight line. The teeth will therefore have a straight line for both faces and flanks, and the line should be inclined so that it is a tangent to the curve of the teeth of the pinion. As in the case of a pair of wheels, the pitch circle of the pinion and the pitch line of the rack must be in contact, or the relative velocity will not be constant, unless the teeth are formed upon the involute principle.

This is the modern practice, and forms the basis of the generation of gears in gear-cutting machines, as will be seen in the later pages of this book. The rack teeth are formed at two standard included angles: 29 deg., which is the angle of the Acme thread, and 45 deg. Half these angles,  $14\frac{1}{2}$  deg. or  $22\frac{1}{2}$  deg., are called the Pressure Angle. The effect of weakening undercutting of the flanks with a  $14\frac{1}{2}$  deg. pressure angle is very obvious, but the teeth of the  $22\frac{1}{2}$  deg. gear are much stronger. Pinions having less than 12 teeth are liable to this defect.

## CHAPTER IV

### Bevel Gears

WHEN two shafts are not parallel with one another, conical toothed wheels, called bevel wheels, may be used to transmit the drive. These wheels are difficult to construct so that they will work properly together and maintain the relative velocities of the shafts. If the shafts are at a right angle and the wheels are of equal size, the gears are then called mitre wheels. When planning a pair or train of bevel wheels, first imagine them as cones with smooth surfaces rolling against each other and transmitting the motion by frictional contact (Fig. 27). The relative velocity of cone  $W$  to cone  $P$  will depend upon the relative diameter  $A A$  of cone  $W$  to the diameter  $B B$  of cone  $P$ . If these diameters are equal, cone  $P$  will make one revolution for each revolution of cone  $W$ . If any other diameters, such as  $C C$ ,  $D D$ , are selected, they will be in the same proportion to one another as the large diameter  $A A$  is to  $B B$ . We can imagine a series of such pairs of diameters between the bases and points of the cones, and each pair will bear the same proportion to one another. The entire surfaces, therefore, of the two cones roll together with the proportional velocity of the large circles  $A A$ ,  $B B$ , and the entire surface of each cone forms a pitch surface of that cone. We could thus select any pair of diameters upon which to form the pitch circles of the cones. In practice, the circles formed upon the largest diameters  $A A$  and  $B B$  are selected as the pitch circles. To drive one shaft by the other at any relative number of revolutions, the sizes of the circles forming the bases of the cones are made in proportion to the

gear ratio between the shafts. Thus, if shaft B is to make one complete revolution whilst shaft A makes one revolution, the base A A of cone W will have a diameter equal to the base B B of cone P ; if shaft B is to make one revolution whilst shaft A makes two revolutions, the base of cone P should be designed

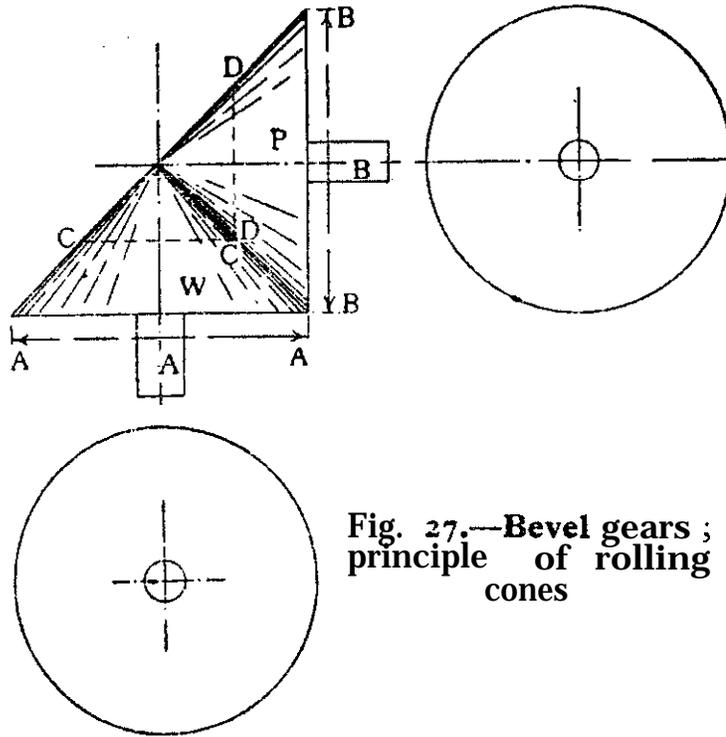


Fig. 27.—Bevel gears ; principle of rolling cones

with a diameter twice as large as the diameter of the base of cone W (Fig. 28). The bases of the cones are equivalent to the pitch circles of flat gear wheels, and the shafts which they connect will rotate with relative velocities proportional to the diameters of the bases of the cones. In these explanations it is assumed that the axes of the shafts intersect, as this is the condition usually met with in practice.

### The Shape of Bevel Gear Teeth

A pair of cones, made of wood, metal, or other material, will transmit the motion of one shaft to the other by contact friction between the surfaces. If these cones are large in proportion to the amount of power to be transmitted, and conditions

of working are favourable, the friction may be sufficient for driving. To prevent slip, teeth may be provided as

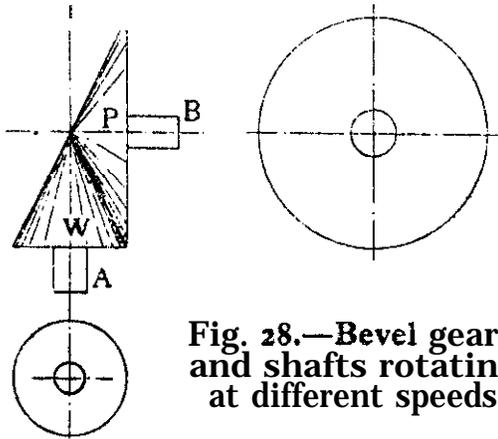


Fig. 28.—Bevel gears and shafts rotating at different speeds

in the case of spur gear wheels. This introduces a difficulty, as the teeth and spaces must be conical and follow the shape of the pitch surfaces of the cones. For example, if we construct teeth of similar shape and character to those used for spur wheels, these teeth

must be made to taper from the base to the point of the cone, as indicated by the shaded surfaces (Fig. 29). If the teeth are made of uniform height and thickness, or of less angle of taper than would terminate in the point of intersection of the pitch cones, they could not work together, but would foul and break off if sufficient power were applied to drive the shafts. Every part of the surface of each tooth—the faces and flanks as well as the tops—must be conical, the taper coming to a point at the intersection point of the pitch cones. The teeth, if properly made, will therefore become very thin near to the points of the pitch cones, finally vanishing away. Only a

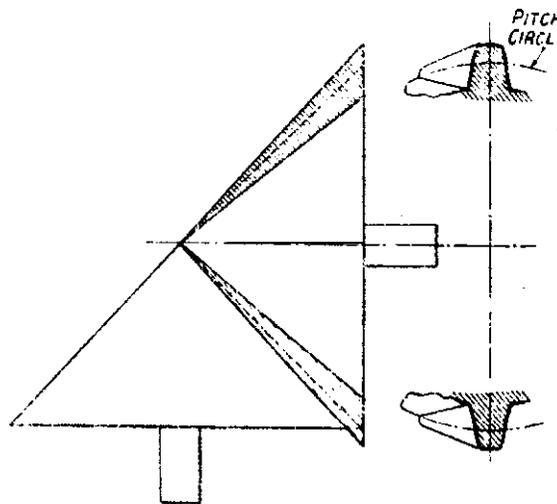
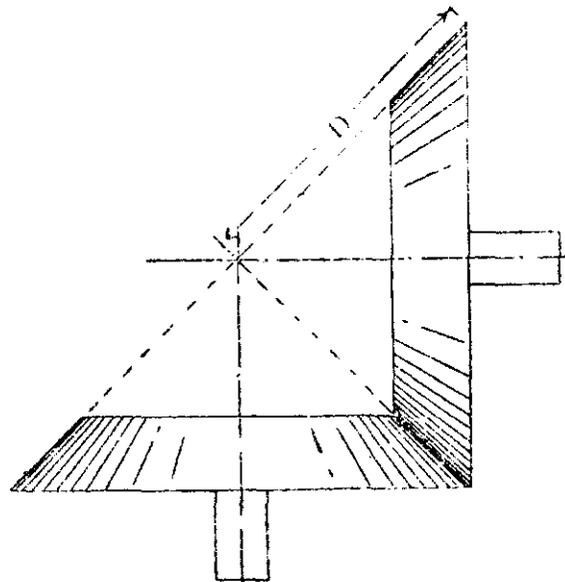


Fig. 29.—Formation of bevel-gear teeth

portion of the tooth length is of practical use, and bevel wheels are never made to the complete theoretical form of the pitch cones; the breadth is usually made equal to one-third the

distance  $D$  (Fig. 30). Each wheel thus becomes a truncated cone, but is actually part of a complete cone, as indicated by the dotted lines (Fig. 30).

Fig. 30.—Practical form of bevel gears

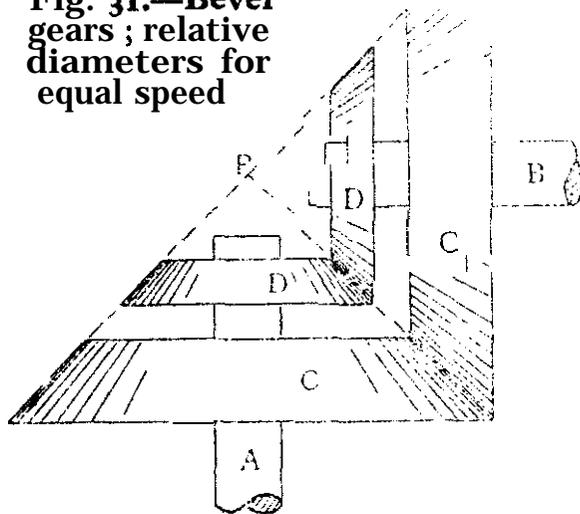


### The Sizes of Bevel Gears

This principle of rolling cones permits considerable latitude in selecting the size of the wheels, and in this respect the problem differs from that of connecting two parallel shafts by spur wheels. In the latter case, the size of the wheels is limited by the distance between the shafts, but when the shafts are at an

angle, the wheels may be of any size within the space available. For example, in Fig. 31 the shafts  $A$  and  $B$  are to be connected by the wheels so that they rotate with equal velocities. Wheels of size  $C$   $C$  may be used, or those of size  $D$   $D$ , or any intermediate size, without affecting the relative speed of the shafts. Both pairs of

Fig. 31.—Bevel gears; relative diameters for equal speed



wheels could be used simultaneously, because all bevel wheels on either shaft, having pitch surfaces meeting on the line of the two

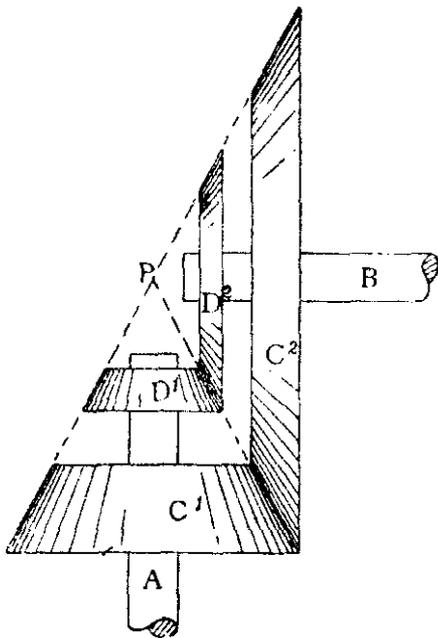
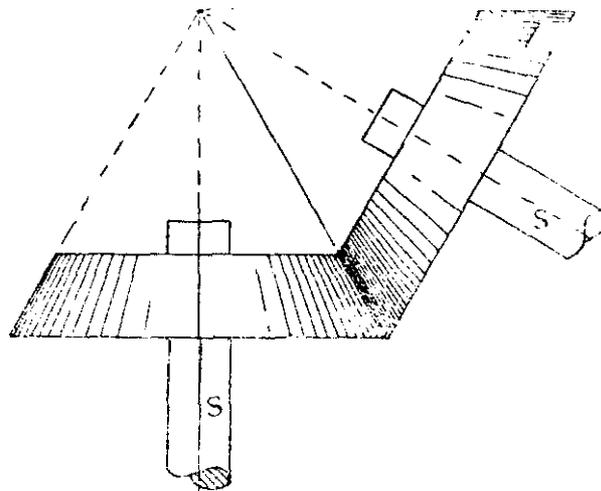


Fig. 32.—Bevel gears ; relative diameters for differing speeds

cones indicated by the dotted lines, are really a portion of one large conical wheel, the teeth and pitch surfaces of which extend from the point to the base of the largest wheel. The principle is not affected if the shafts rotate at different relative velocities. For example, in Fig. 32 shaft A makes two revolutions to one revolution of B. The wheels may be of size C C or D D, or any other size, provided their pitch surfaces form part of the cones indicated by the dotted lines. The relative numbers of teeth must remain the same or

be in the same ratio. If C1 has 30 teeth and C2 60 teeth, D1 must have 30 teeth, and D2 60 teeth, or numbers of teeth having a ratio of 1 to 2 ; thus D1 could have 15 and D2 30 teeth, and so on. Obvi-

Fig. 33.—Principle of rolling cones ; shafts placed at less than 90 deg.



ously, a large pair of wheels can have teeth of greater size and strength than a smaller pair. When the shafts are not at a right angle to one another the principle of rolling cones is still applicable, if the axes of the shafts intersect. Fig. 33 is a diagram showing two shafts intersecting at an angle of less than 90 deg., and Fig. 34 shows the shafts intersecting at an angle greater than 90 deg. ;

in each instance, the cones have equal diameters, so that the two shafts will rotate at equal speeds. The shafts may be made to rotate at different speeds by designing the cones so that their diameters are of corresponding proportions to the speeds, as in the case of shafts at a right angle. Fig. 35 shows the principle of rolling cones applied to an internal gear connecting two shafts

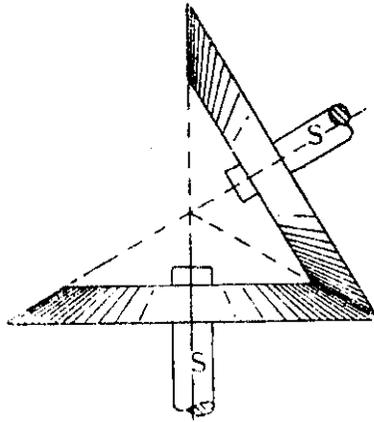


Fig. 34.—Rolling cones : shafts placed at angle greater than 90 deg.

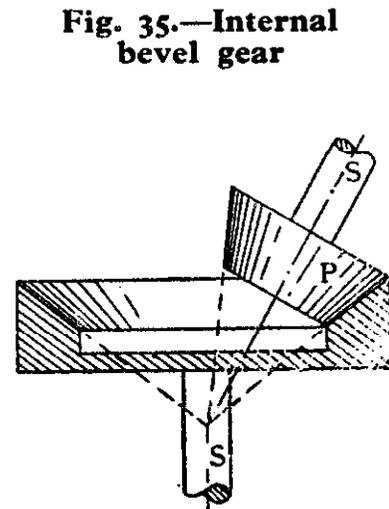


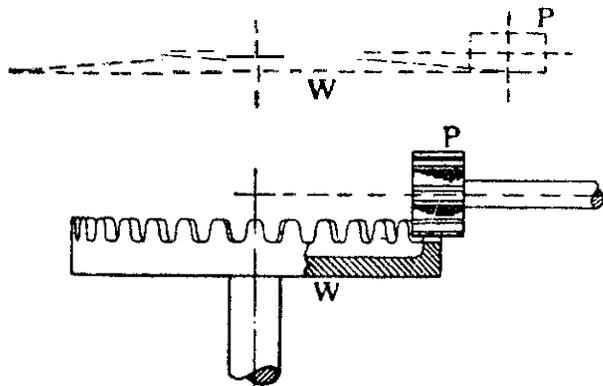
Fig. 35.—Internal bevel gear

S, S, which are at an angle. In this arrangement the wheel P must be smaller than the other, as it is a pinion working inside an annular wheel.

### Crown Wheel and Pinion

The term crown wheel and pinion is sometimes given to the gear shown in Fig. 36. Correctly speaking, crown wheel is another name for bevel wheel, and the gear shown in Fig. 36 should be formed on the principle of rolling cones. If the wheel W is made with straight teeth and a cylindrical pinion P is used to gear with it, the arrangement will not work correctly. The wheel may be represented by a flat disc W (Fig. 37), and its pitch surface would be a part of the disc. The pinion would be represented by a cylinder P, rotating in contact with the disc ; such a cylinder would form the pitch surface of the pinion.

Obviously all parts of the circumference of P will move with the same velocity. But all parts of the surface of the disc



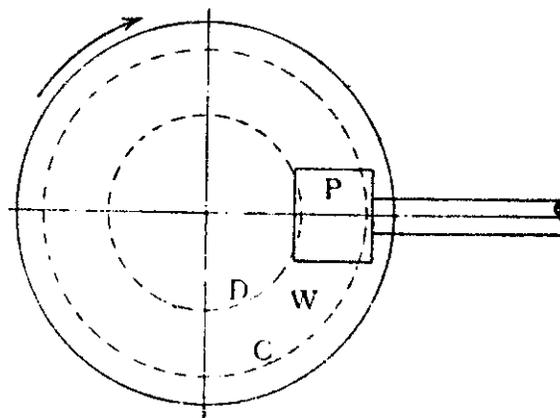
will not move with the same velocity. That part represented by the dotted circle C

Fig. 36.—Crown wheel and pinion

will have a much greater surface speed than the part represented by the dotted circle D. As both parts

are in contact with the circumference of P, the circle D will be trying to drive P at a slower speed than it is being driven by P. Every part of the surface between C and D will therefore be trying to rotate P at a different rate of speed. As P can only rotate at one speed at any instant, a slipping and grinding action must take place between the surfaces. If the surface of the disc is cut away so that only a circular ridge is left in contact with P, such as would be represented by the circle C, this action would be reduced to a minimum. Therefore the gear shown in Fig.

Fig. 37.—Emphasising narrowness of crown wheel teeth



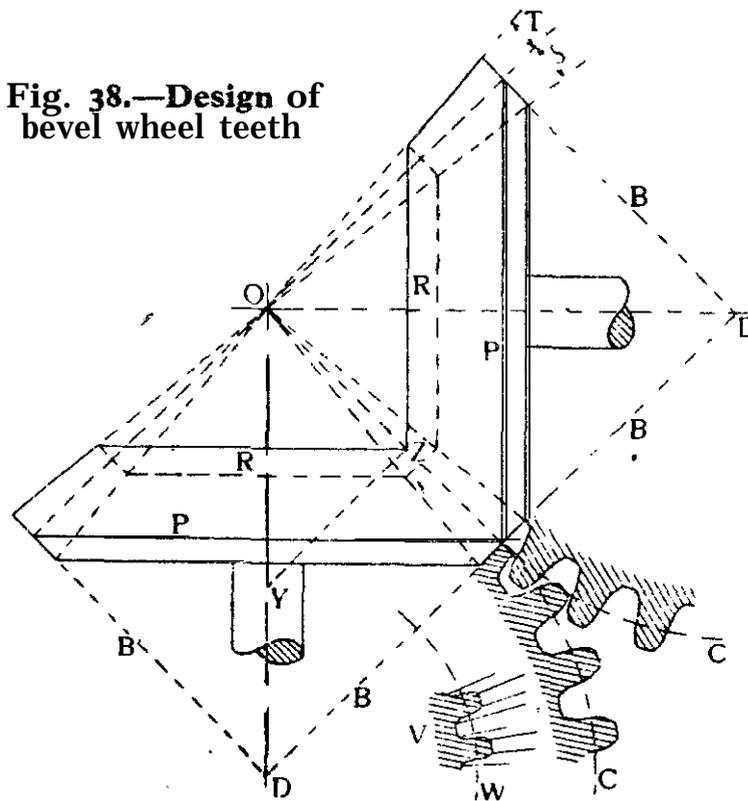
36 can be made to work if the teeth on W have very small breadth, as indicated by the sketch, so that they make very narrow contact with P. Such a wheel can then only transmit or receive a very small amount of power or its teeth will soon wear away. When the diameter of the wheel is large compared to that of the pinion, the error in the shape of the teeth as regards

taper is small, as indicated by the dotted sketch Fig. 36, but it still exists.

### The Shape of Bevel Wheel Teeth

The teeth of bevel wheels are formed on the same principle as the teeth of spur wheels, but the shapes of the faces and flanks are not developed upon the actual pitch circles. A section through a tooth which would show its actual shape would not be in a plane parallel to the base of the pitch cone, as indicated

Fig. 38.—Design of bevel wheel teeth



by the teeth shown by Fig. 29, but would be in a plane perpendicular to the conical pitch surface. The teeth are placed so that they are perpendicular to this pitch surface; therefore, it would not be correct to develop their shape by curves generated on the circumferences of the pitch circles. They are developed upon circles  $C, C'$  of larger diameter, as indicated in Fig. 38, the centres and radii being found by drawing lines  $B$  at a right angle to the pitch surfaces, and meeting the centres of the shafts at  $D, D'$ .

## CHAPTER V

### Worm Gears

TYPE of gearing, which is frequently adopted when the driven wheel is required to give a much lower number of revolutions in a given interval of time than the driver, is shown in Fig. 39. The arrangement is called worm gearing. The driving wheel is a screw *S*, and is called the worm. The driven wheel *W* is provided with teeth, and is called a worm wheel. Imagine the wheel *W* to be fixed so that it cannot rotate. If the worm, *S*, is rotated, and can move also in a direction along the line of its axis, it will act as if *W* was a nut through which it was being screwed, because the thread of the worm is engaging with the teeth of the wheel. The worm will therefore move in a forward or backward direction, depending upon the direction in which it is being rotated.

If, on the contrary, the shaft of the worm is held between thrust bearings so that it cannot move in an endwise direction, and the wheel *W* is free to rotate, it will do so if the worm is rotated. As the worm is unable to screw itself past the wheel, the latter will rotate owing to the sliding action of the worm thread upon the wheel teeth. The rotation of the shaft *S* will be thus transmitted to the shaft upon which *W* is fixed.

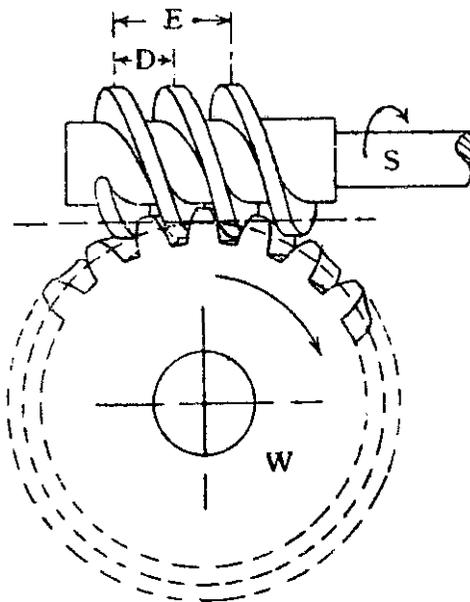


Fig. 39.—Worm gearing

### Worm and Spur Gearing Compared

Worm gearing, though equivalent to a pair of spur wheels in its action, differs to some extent. Either wheel of a pair of spur wheels may be made to drive the other, but though the worm can always be made to be the driver, the wheel will not necessarily drive the worm. As in the case of spur wheels, the gear is designed in accordance with pitch lines and surfaces. There is this difference, however—the pitch surfaces of spur wheels roll together and, as already explained, one would drive the other by contact friction if the load upon the driven wheel were not excessive. The pitch surfaces of the worm gear (Fig. 39) are represented in Fig. 40; obviously, if S is rotated its effort will be expended entirely in a line parallel to the shaft of W, and will not produce any rotating effect on W. The surfaces will merely grind together without producing any turning effort upon W. Similarly, if W is rotated, the effort will be expended entirely in a line with the shaft of S, and no rotating effect will be produced. Any rotary effort can therefore only be produced by providing S and W with teeth which are placed at an angle to the axes of the shafts and can slide against one another. This is effected in practice by means of a screw thread upon S and teeth upon W, which are machined at an angle to correspond with the inclination of the screw so that the two will engage in gear. The amount of rotation which will be given to W for each complete revolution of S will therefore depend upon the pitch of the screw thread. The pitch, divided into the circumference of the pitch circle of the worm wheel, gives the number of revolutions which the worm will make to produce one complete revolution of the wheel.

### Multiple-thread Worms

The relative number of revolutions made by S and W is therefore quite independent of the diameter of the pitch surface of S, and is also, in a sense, independent of the number of teeth upon W. For example, suppose the screw S to have a single

thread of 1-in. pitch and the pitch circle of the wheel *W* to have a circumference of 20 in. The wheel should then have twenty teeth of 1-in. circular pitch. As the pitch of a screw is the distance through which the thread advances whilst making one complete turn round its axis, *S* must make twenty complete revolutions to drive *W* through one complete revolution. If the screw thread is made to have a pitch of 2 in., the wheel *W* would then be made to have ten teeth of

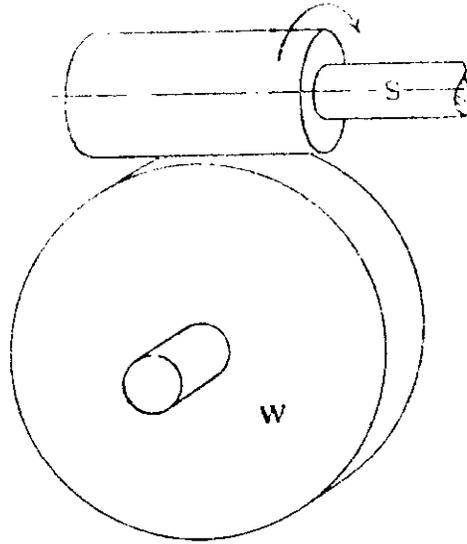


Fig. 40.—Worm gearing: principle of pitch surfaces

2-in. circular pitch. The screw would then make ten revolutions to drive *W* through one complete revolution. So far, the number of teeth on *W* has been made proportional to the ratio of the gear, that is, we have halved the number of teeth whilst obtaining half the number of revolutions of *S* required to obtain one complete revolution of *W*. But we need not have reduced the number of teeth on *W*. We could have allowed the screw to gear into alternate teeth, half of the number of teeth thus being unused. The arrangement would effect the desired result as, for each revolution, the screw would move the circumference of *W* twice as far as with a screw pitch of 1 in. The screw would thus give one revolution to *W* when its own shaft had made ten instead of twenty turns. But it would not be a good arrangement to permit half of the number of teeth to be idle. The whole twenty teeth can be utilised by providing a second thread upon *S*, interspaced with the first thread, to gear with the idle teeth. Each thread will then take a share in driving the wheel, and the pressure and wear will be distributed over double the amount of contact surface. It would be necessary for the teeth to be re-shaped to fit the altered curve of the screw

due to the increased pitch. The wheel *W* thus retains its previous number of teeth, namely, twenty, and yet makes one revolution for every ten instead of every twenty revolutions of the screw. Similarly, the screw may have three or more threads. The number of teeth upon the wheel therefore does not determine the ratio of the gear which, as already stated, is determined by the pitch of the worm.

When considering worm gearing, it is therefore advisable to call the pitch of the *worm* the lead, as it will differ from the circular pitch of the teeth on the *wheel* if the worm has more than one thread. The term "lead" also expresses more correctly the function of the worm thread in its relation to the wheel, as represented in Fig. 39, if the worm has a single thread its pitch will be the distance *D*, and this will be equal to the circular pitch of the teeth on *W*. But if the worm has a double thread, its pitch will be the distance *E*; this will be twice the length of the circular pitch of the teeth on *W*.

### Involute Worm Wheel Teeth

When preparing the worm blank and wheel blank for machining, allowance must be made for the distance beyond the pitch line by which both the thread of the worm and teeth of the wheel will project. If a section be taken through the centre line of the worm, the teeth of the wheel and the thread of the worm can be regarded as a pinion and rack at that line. The screw thread then represents the rack, and it is designed in accordance with the method used for determining the shapes of the teeth of a pinion and rack. These teeth may be curved upon the cycloidal or involute principle. If the former method be used, both the teeth and screw thread will have curved sides; if the involute method be adopted, the sides of the screw thread will be straight lines, as already explained in an earlier chapter. The involute principle is usually adopted, because it is easier to cut the worm thread if it has straight sides. Fig. 4 1 shows a section of a worm gear through the centre line *L M*.

### Cutting Worm Wheels with a Hob

If the teeth of the wheel accurately fit the spaces between the thread of the worm throughout the entire breadth of the wheel and follow the true curve of the screw, their shape will alter in section according as the distance from the centre line  $L M$  is increased. A section of any tooth taken on any line except  $L M$  will show a different shape to that taken on line  $L M$ . In addition to this, the circumference of the wheel must be hollowed

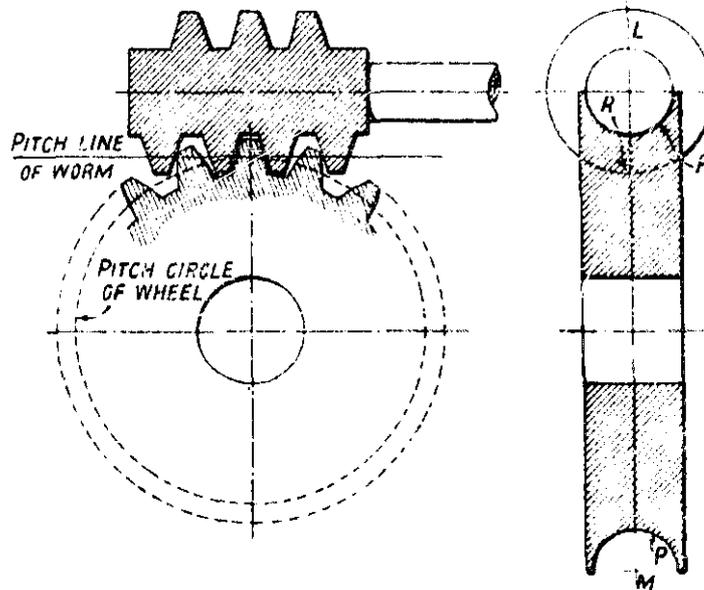


Fig. 41.—Worm gearing : shapes of wheel teeth and worm thread

to fit the worm at the points of the teeth and bottoms of the spaces between the teeth, the curves being arcs of circles of two different radii, as indicated by  $P$  and  $R$  (Fig. 41). On this account, shaping and cutting the teeth of a worm wheel correctly is a difficult matter. The method adopted in practice, especially for wheels of small or comparatively small sizes, is to shape them by means of a cutter which is a facsimile of the screw intended to gear with the wheel. This cutter is called a hob, and consists of a steel worm of exactly the shape of the worm which is to gear with the wheel; it is provided with cutting edges and hardened. The teeth of the wheel are first cut nearly to size

by means of an ordinary circular cutter ; the hob is then geared with the wheel, and the two are run together until the hob has cut the teeth to the true shape. Obviously, if the hob is a correct representation of the worm it will remove all irregular places from the teeth and leave them a perfect fit to the actual worm. It is sufficient, therefore, to plan the thread and teeth upon a single section L M taken through the centre line of the worm and wheel.

### Other Methods of Cutting Worm Wheel Teeth

A correctly shaped worm wheel will have the appearance of

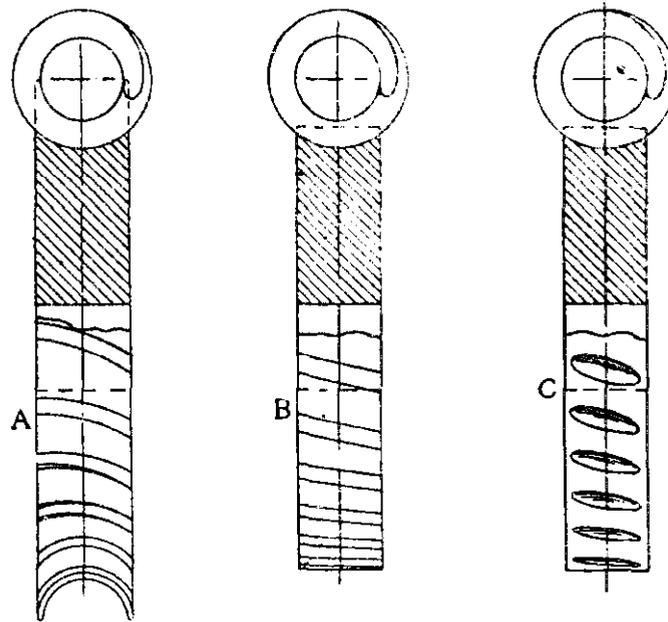


Fig. 42.—Shapes of worm wheel

A (Fig. 42). On account of the expense of making a hob, the form of worm gears is often a compromise. The circumference of the wheel is not hollowed at all, but straight, as in the case of an ordinary flat spur wheel. The teeth are cut on the slant, as indicated by B (Fig. 42), and at an angle to correspond with the inclination of the worm thread. Another method is to make the circumference of the wheel straight and to cut the teeth

with a circular milling cutter, as indicated by C (Fig. 42). If the axis of the worm is not at a right angle to that of the wheel, an ordinary flat spur wheel can be used by slanting the worm until its thread meshes with the teeth of the wheel, as indicated by Fig. 43. Any one of these methods (Figs. 42 and 43) may be used successfully, and for the transmission of a very small amount of power the worm can be an ordinary Whitworth or similar screw thread. In practice, the edges of the wheel are usually bevelled off, as indicated at C (Fig. 44), except in the case of wheels like C (Fig. 42). This diminishes the

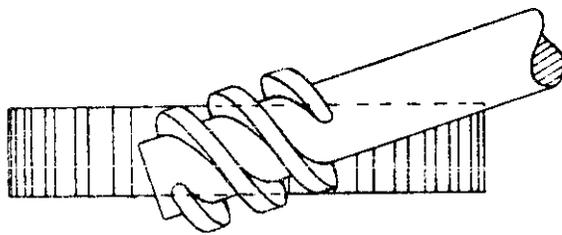


Fig. 43.—Worm wheel with straight teeth

inaccurate portion of the teeth and removes the weak corners. A blank worm and wheel, prepared for cutting a gear such as Fig.

39, would have the appearance indicated in Fig. 45 if the teeth were to be of perfect form and shaped by means of a hob. The shaped by means of a hob. The dotted lines show the pitch lines and the allowance of metal required to form the part of the teeth projecting beyond the pitch line. The part P is called the "throat," and should be curved to a circle equal to the diameter of the worm at the bottom of its thread. As a small amount of clearance should exist between the worm and wheel at the tops of the teeth and bottom of the spaces, the hob should be made slightly larger in diameter than the worm, so that it will produce this clearance. Fig. 46 shows a blank wheel similar to Fig. 45, but which is to be cut with straight-through teeth, as B (Fig. 42). The throat P is now made straight, and not curved, as in Fig. 45.

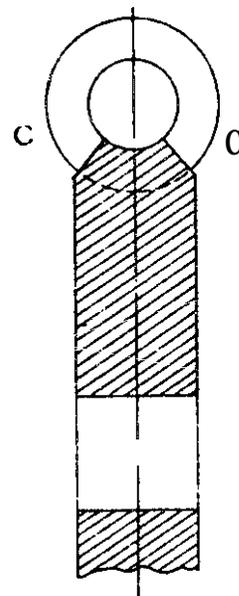


Fig. 14.—Practical form of worm wheel

### Gear Ratios and The Worm Lead

The diameter of the worm has no influence upon the ratio of the gearing. As already explained, this is determined by the lead of the worm thread and the circumference or the diameter of the wheel. If the distance between the shaft centres of worm and wheel is fixed, you must select a lead for the worm thread that will give the ratio desired. The diameter of the worm can be made greater or less to accommodate the wheel selected. For example, suppose the distance between the centres will admit a wheel having a circumference of 10 in. ; the pitch circle for this will have a diameter of  $3\frac{3}{16}$  in. If the worm thread is made with a lead of one turn in  $\frac{1}{2}$  in. ( $\frac{1}{2}$ -in pitch), the ratio of the gear will be 1 to 20, because each revolution of the worm will rotate the wheel by  $\frac{1}{2}$  in. As there are twenty half-inches in the circumference of the wheel, twenty revolutions of the worm are required to rotate the wheel through a distance of 10 in. If a ratio of 1 to 10 be required, it can be obtained by making the worm thread with a lead of one turn in 1 in. Ten revolutions of the worm will then produce one revolution of the wheel. If a greater ratio than 1 to 20 be required, it can be obtained by decreasing the lead of the worm thread. For example, if the lead is one turn in  $\frac{1}{4}$  in., the ratio of the above gear will be 1 to 40, as there are forty  $\frac{1}{4}$  in. in a circumference of 10 in. ; therefore, forty revolutions of the worm will be required to produce one revolution of the wheel. If the worm thread is made with a lead of  $\frac{1}{10}$  in., the ratio of the gear will be 1 to 100. The lead may be one turn in 2 in. ; the ratio will then be 1 to 5. As the lead of the worm thread is increased, the thickness and height of the wheel teeth must be increased also. It may therefore be necessary to cut several threads upon the worm to enable the teeth to be made of reasonable proportions, as previously explained. In the instance given above, where the worm thread has a lead of 2 in., it may be advisable to cut four or five threads. The wheel would then have twenty or twenty-five teeth respectively. Should the circumference of the wheel pitch circle, a s first

determined, prove not to contain the exact number of teeth, it may be increased or decreased to a limited extent and the

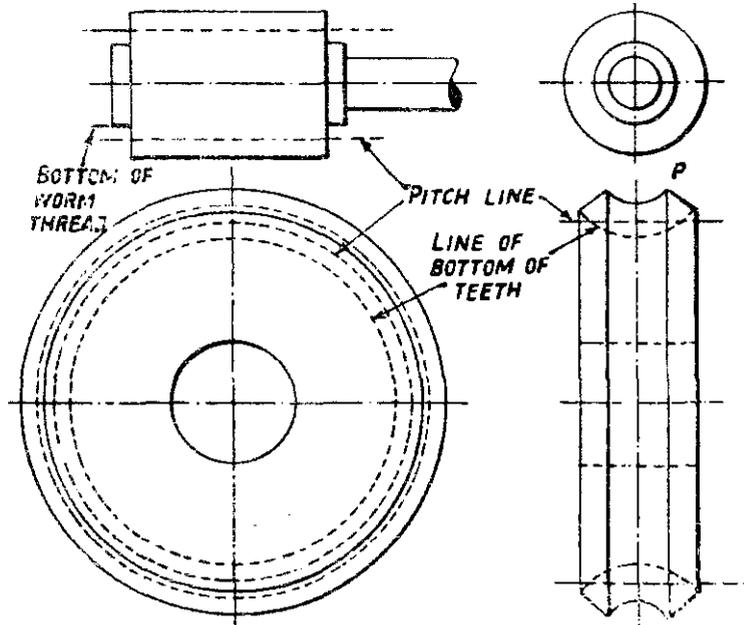


Fig. 45.—Determination of blanks for worm and wheel

diameter of the worm altered to make good the discrepancy. When the centres are not fixed, the wheel size is only limited by constructional requirements. It is, however, advisable to

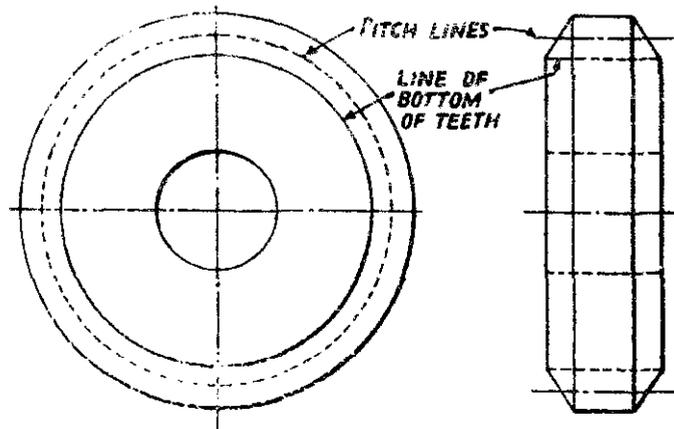


Fig. 46.-Blank for worm wheel with straight-through teeth

have at least thirty teeth in the wheel. When a smaller number must be used, the tips of the worm thread will not properly

clear the wheel teeth. This interference can be avoided by a slight rounding off towards the tops of the thread, or by increasing the diameter of the wheel, so that the teeth project almost entirely outside the pitch circle. Messrs. Brown and Sharpe, in their treatise on gearing, give the following rule for this increase of diameter. The pitch diameters to be multiplied by .937; add to the product four times the addendum, that is, the part which in the ordinary way would be outside the pitch circle. The sum gives the diameter of the blank at the throat P (Fig. 46). The whole diameter of the wheel is obtained by making a drawing to this rule and measuring off the dimension. Messrs. Brown and Sharpe say, however, that it is not practicable to finish wheels, sized by this rule, with a hob when they have twelve to eighteen teeth; unless the wheel is driven by a separate mechanism the hob must not be relied upon to drive the wheel.

### Proportions and Efficiency of Worm Gearing

Professor Unwin gives the following proportions for worm gearing,  $P$  being the circumferential pitch of the wheel teeth. Thickness of tooth on pitch line,  $.48 p$ ; height outside pitch line,  $.3 p$ ; depth below pitch line,  $.4 p$ ; length of worm, 3 to 6  $p$  (usually 4  $p$ ); width of wheel face, 1.5 to 2.5  $p$ . The worm is frequently made of some different metal to that used for the wheel. For example, a steel worm and gun-metal wheel, a hardened steel worm and a phosphor-bronze wheel give good results; a wrought-iron or steel worm and a cast-iron wheel are also used; a cast-iron worm can be used with a cast-iron wheel. When the gear is used for continual running and transmitting power for driving purposes, the shape and materials are of much greater importance than when the gear serves only for adjustment purposes and occasional use. A hardened steel worm and a phosphor-bronze wheel are a very good combination for transmitting power, but efficient lubrication of the contact surfaces is the most important factor. The gear should run in an oil bath if possible. Worm gearing was at one time regarded as a very

inefficient means of transmitting power, but during recent years it has come into extensive use for this purpose and, if well designed and run in oil, it is found to have a high efficiency. The maximum so far recorded, we believe, is still 97.3 per cent., with D.B.S. glass, when tested at the National Physical Laboratory. The loss of power due to friction between the worm thread and wheel teeth decreases with increase of the inclination of the thread ; that is, a coarse lead will give a higher efficiency than a fine lead pitch. Worms having multiple threads thus give a higher efficiency than single-threaded worms, and a small diameter worm gives a higher efficiency than one of corresponding lead, but of larger diameter. In addition to the friction at the worm thread, there is friction set up by the end-thrust on the worm shaft. This is also of importance, and some form of thrust

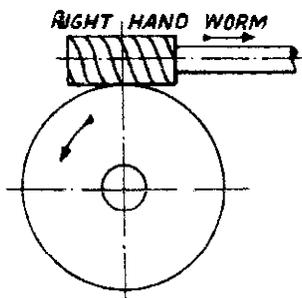


Fig. 47.—Right-hand worm

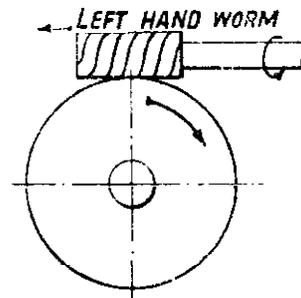


Fig. 48.—Left-hand worm

Relative motions

bearing is required if high efficiency is to be maintained; a ball thrust bearing is, here, advisable. As previously explained, the worm must be prevented from moving end-wise, if it is to exert pressure upon the wheel teeth and rotate the wheel ; therefore, the wheel teeth will press against the worm thread with a force proportional to the driving load. The teeth, therefore, thrust the worm shaft against the bearing in which it runs. The direction in which the wheel rotates for a given direction of rotation of the worm depends upon whether the worm thread is right- or left-handed. It is possible on this account to combine two worm gears so that the end thrusts of the worms oppose each other, and no thrust is imposed on the bearings.

Fig. 47 is a diagram of a worm and wheel, in which the thread is right-handed, and Fig. 48 is a similar design, in which the thread is left-handed. The arrows indicate the direction of rotation and thrust. The worm shaft rotates in the same direction in either instance, but the wheels rotate in opposite directions, and the direction of thrust exerted upon the thread of the worm, and therefore upon its bearings, is also opposite, as indicated by the straight arrows. In Fig. 49 the gears are combined, and the two worms are cut on one shaft. Each worm exerts a thrust in the opposite direction to that produced by the other; the bearings are thus relieved of end pressure, which is taken by the part of the shaft connecting the two worms.

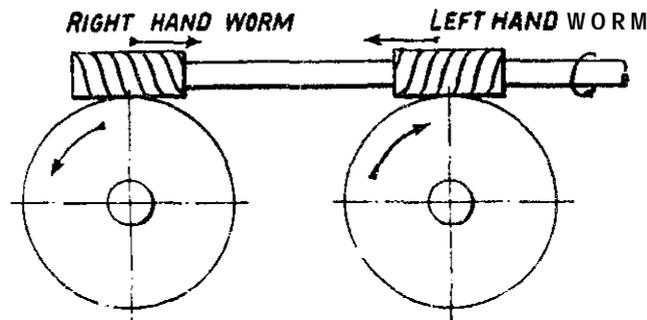


Fig. 49.—Neutralising end thrust of worm gearing

The driving force of the wheels can be combined and transmitted to a single shaft by means of spur wheels.

A distinctive feature of worm gearing is that it is not always reciprocal; that is, the worm will always drive the wheel, but the wheel may not drive the worm. If the lead of the worm is small, and therefore its angle small, the friction between the surfaces in contact will be so great that the worm cannot be rotated by the wheel. The critical lead of thread to enable the wheel can drive the worm will depend upon the friction between the surfaces in any particular instance. Generally speaking, single-thread worms cannot be driven by the wheel, but if the lead required is sufficiently great to necessitate a multiple thread, the wheel may drive the worm; the greater the lead, the more likely is the gear to be reversible.

## CHAPTER VI

### Special Forms of Gear Wheels

#### Helical Gear Wheels

IT should be noted that with straight-toothed gear wheels, that is, those having teeth parallel to the axis of the wheel, the teeth when in motion slide upon one another. The pitch circles roll together, but the teeth rub and slide upon one another as they enter into and leave engagement, resulting in wear and tending to produce noise when running at high speed. To minimise this, "stepped gearing" was devised, and is applicable both to wheels and racks and pinions. Imagine several thin gear wheels placed side by side on a shaft, but the teeth, instead of being set in a straight line, are placed so that they form a series of steps (Fig. 50). If the wheels are fastened together they will form one gear wheel with stepped teeth. If such a wheel is geared with another similar wheel, the load transmitted by the driving wheel to the driven wheel will be applied progressively instead of suddenly as when straight teeth are used. Obviously, with a given breadth of wheel, the greater the number of steps the more gradual and smoother will be the application of the driving force. If the number of steps is made infinitely great the steps vanish and become merged into a curved line, and the application of the force will therefore be infinitely gradual. Actually, the

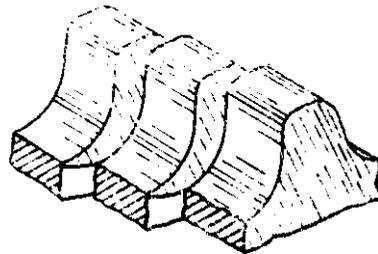
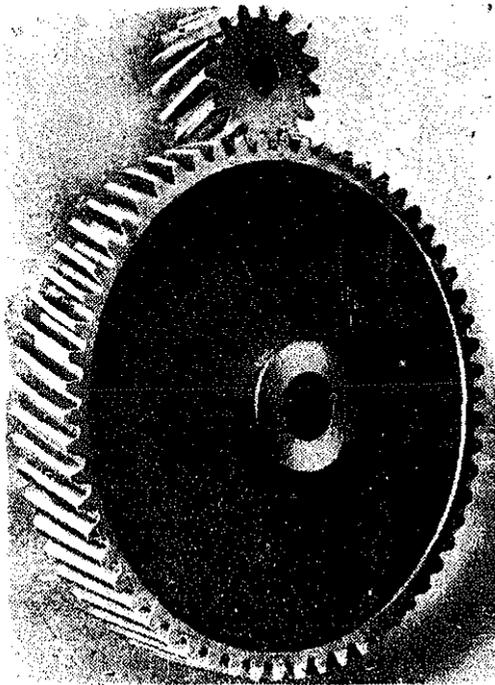
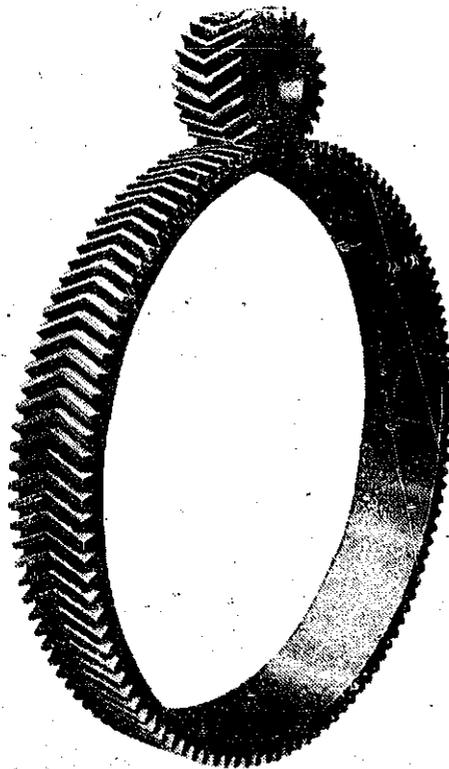


Fig. 50.—Stepped teeth



**Fig. 51.—Single helical gear wheel**  
*By courtesy of David Brown & Sons*  
*(Huddersfield) Ltd.*



**Fig. 52.—Double helical gear**  
*By courtesy of David Brown & Sons*  
*(Huddersfield) Ltd.*

curved line becomes a helix or screw thread, hence the term "helical" applied to this method of constructing gear-wheel teeth (Fig. 51). In action, the sliding of the teeth is abolished and replaced by an effect of pressure and rolling only. A practical advantage is that, with a wheel-and-pinion drive, a pinion of smaller diameter can be used with good results; there is also diminution of shock at high speed, and with an uneven load, there is less vibration and noise, and at the same time the efficiency of the transmission is also increased. The curves for any pair of wheels in gear must be of opposite hands; that is, if the driver, for example, has its teeth on the curve of a right-hand helix, those of the driven wheel must be on the curve of a left-hand helix and, further, the angle or slope must be the same in each wheel, see Fig. 51. There is a disadvantage in the use of helical teeth, for the conversion of the sliding action, which obtains with straight teeth, into a rolling pressure action results in an endwise

thrust upon the shaft ; in effect the wheels try to push themselves apart sideways. On this account " single " helical teeth, as they are termed, are not suitable if a heavy driving force is to be transmitted. If, however, a second pair of wheels, with teeth sloped so that the endwise thrust is in the opposite direction to that exerted through the first pair, is placed upon the shafts, the two thrust effects will be in opposition and there will be no endwise thrust upon the shafts or bearings. The endwise thrust of one pair of wheels will neutralise that given by the other pair, whatever may be the amount of variation in force applied through the gearing. Instead of having two pairs of wheels, the two drivers and the two driven wheels might respectively be fastened together, so as to form one driver and one driven wheel. Usually, in

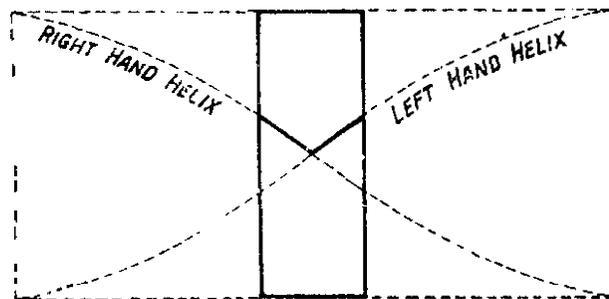


Fig. 53.—Derivation of curves of helical teeth

practice, this is effected by making each wheel as one, the teeth being then termed " double helical " (Fig. 52). This type of gearing is much used in mills and factories where heavy duty is required. To visualise the formation of the curved teeth in a double-helical gear wheel imagine that the wheel is a slice cut from the middle of a cylinder upon which are traced two helices of screw threads (Fig. 53), one being left hand and the other right hand. The portions of the curves near to the point of intersection represent the curves of one tooth. Every tooth in the wheel is composed of portions of two similar curves. Double or single helical teeth are applied to bevel wheels and pinions, and also to racks and pinions. The teeth of a double-helical wheel are not always placed so that the two portions meet ; these are often " staggered," that is, one row is halfway in advance of the other

row. Helical teeth are made by the processes of cutting or casting, special appliances being used.

### Screw or Spiral Gear Wheels

These are another special kind of gear wheel having screw or spiral teeth, but the term is particularly applied when the axes or shafts are not in the same plane ; for example, if the shafts cross each other at a right angle and at some distance apart. The shafts may, however, be at any angle to one another. When the shafts cross at a right angle the teeth of each wheel must be the same hand, that is, they are curved to a screw or helical line which must be right-handed or left-handed in both wheels. Whether the curve is to be right- or left-handed depends upon the direction of motion required between the respective shafts. Fig. 54 shows an example of spiral gear wheels. Strictly, the term screw gearing or helical gearing includes all the forms in which the teeth are made with a helical or screw curve, but in practice these are divided into worm gearing, helical gearing with single or double teeth, and spiral gearing. The subject is wide and involved and various arrangements of teeth have been devised and adopted for use under special names, but the principles as explained in Chapters I to IV apply to them all.

### Chain Gearing

Transmission of power and motion through gearing from one shaft to another can be effected by a combination of toothed wheels and chains ; this is termed chain gearing. Imagine a pair of toothed gear wheels fixed upon parallel shafts and each engaging with a rack which is free to move in the direction of its length. If one of these wheels is made a driver and power is applied to it, the rack will be moved along and will rotate the other wheel. Obviously, if the transmission of power and motion is to be continuous and in one direction, the rack must be of a length too great to be practicable. If, however, the rack is made flexible and is carried round the wheels and joined at its ends, the motion may be continued indefinitely. Conceivably, such a flexible rack might be made ; in practice, it is resolved into links

and becomes a chain ; therefore a gear chain may be conveniently regarded as an endless flexible rack. The principles which apply to wheel and rack gearing, as explained in Chapters I to III, form the basis upon which chain gearing is designed. The distance between the pins of the chain must be of some definite pitch, similar to that of the teeth or pins of a rack ; such chains are therefore termed " pitch chains," as distinct from the ordinary hoisting chain working on a smooth drum. Sometimes notches or recesses into which the links of this chain engage are provided upon the pulley to prevent slip, but these are entirely

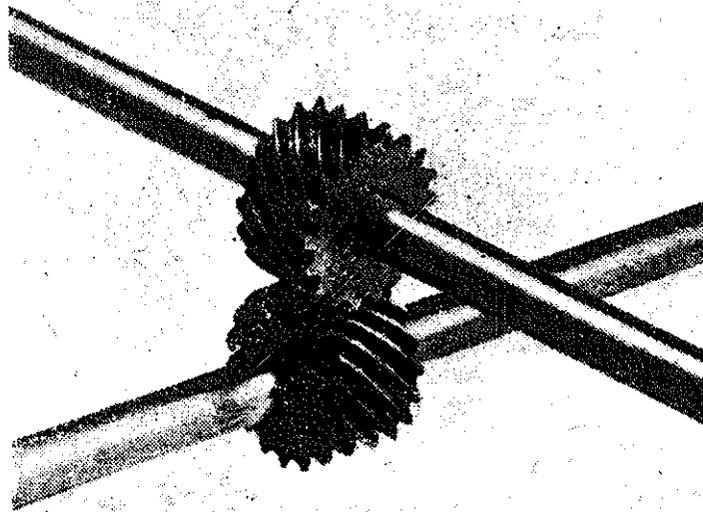


Fig. 54.—Spiral gear wheels

for the purpose of obtaining a sure grip and not for the accurate transmission of motion.

#### Forms of Pitch Chains

There are various forms of pitch chains. The familiar cycle chain has links consisting of flat plates connected by studs or pins upon which rollers or sleeves are mounted (Fig. 55). The teeth of the chain wheels fit between the sides of the links and press against the pins. In action, there is no relative movement between the teeth of the wheel and the pins or rollers of the

chain whilst they are in contact, except at the start and end of engagement. The teeth and pins do not slide upon one another as do those of gear wheels and a rack and pinion. The teeth of the wheel therefore need only be made of a shape which will permit them to enter and leave the links without interfering with the engagement of the pins. Owing to the links being rigid, however, the curve is an arc of a circle struck from the centre of the pin of the link preceding the one just leaving engagement, or, at the engaging side, from the centre of the pin of the link in advance of that just coming into engagement. The general shape of the teeth, therefore, is composed of these two arcs joined at the root by another portion of a circle which corresponds to the periphery of one of the pins or rollers.

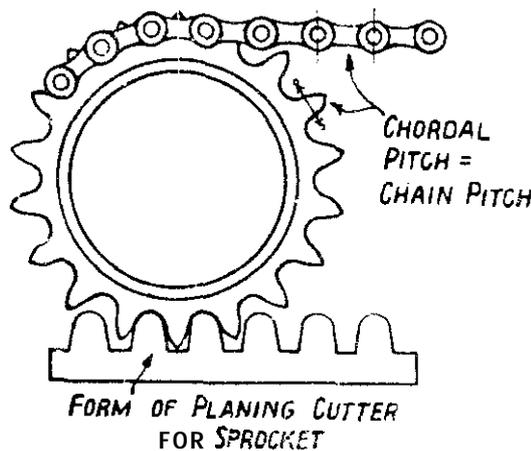
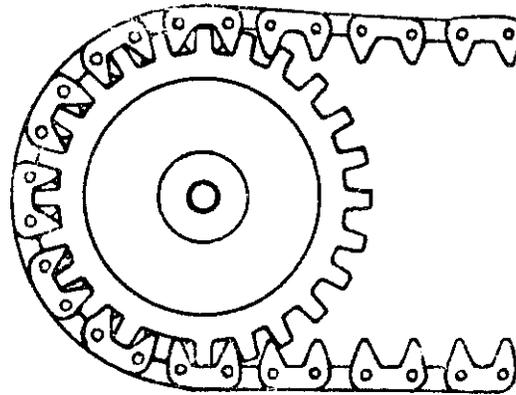


Fig. 55.—Chain and sprocket drive

The points of the teeth may be sharp or blunt according to the general design of the wheel and chain ; the essential consideration is that they clear the pins as the latter come into and leave engagement. In practice, the teeth of chain wheels are shaped according to the experience of the maker to suit practical conditions of service.

Whilst the pins and teeth are in contact, there is pressure between them tending to crush in the surface ; there may also be grit due to conditions of working, and there is slight movement between them at moments of engagement and disengagement. The teeth, therefore, tend to wear away and become undercut at the roots. The pitch of the chain tends to lengthen, due partly to wear of the journals of the pins or bushes, and possibly by actual stretch of the side plates. Makers of chain wheels therefore endeavour to shape the teeth so that they will accommodate to some extent the effect of wear and stretch in

the chain. Frequently, straight lines are substituted for arcs of circles at the faces of the teeth, but this is a matter which depends upon the opinion of the maker. Another type of pitch chain is used for transmission of power instead of belting ; this will work at higher speeds than would be sound practice with open link chains.



*INVERTED TOOTH CHAIN*

Fig. 56.—Chain drive with inverted teeth

Several constructions have been invented and patented, the terms silent chain, high-speed pitch chain or certain proprietary names, being applied to distinguish them. These three chains have, as shown in Fig. 56, inwardly projecting teeth, which engage with the teeth of the wheel, both being shaped to the ideas of the particular maker, so as to comply with the condition that the teeth engage and disengage with freedom. The chain is usually of considerable width as compared with an open-link chain, and the links consist of a number of thin plates connected by pin joints. The links are designed so that the effects of wear are automatically neutralised. Chain gear wheels are termed sprocket wheels.

### **Epicyclic Gearing**

When there are two or more wheels in gear to transmit motion, the arrangement is termed a "train." If the wheels are all in line with one another it is termed a simple train ; if the motion is transmitted through wheels placed side by side upon one or more of the shafts the gearing is termed a compound train. Arrangements of gear wheels in trains form a section of "Kinematics" (the science of movement) ; this is too wide a subject to be dealt with in this book except in a merely introductory manner. Included in this subject is a section relating to Epicyclic

Trains. A brief mention and explanation is given here in order that the reader may be familiar with the term "epicyclic" as applied to gearing. An ordinary train of gear wheels in certain arrangements may easily be mistaken for an epicyclic gear unless the observer understands the basic underlying principle.

In an ordinary train of gear wheels the motion is derived by one or other of the wheels being rotated about its axis. In an epicyclic train the motion is derived by rotating a bar or arm, carrying at least one of the wheels, around the axis of another wheel of the train. The result of this is that the wheels receive a compound rotation; in addition to the amount due to the value of the train, one revolution is given due to the rotation of the bar. The epicyclic principle may be applied to external and to internal gears, and there may be external rotary motion applied to one or more of the wheels in addition to that derived from the rotation of the arm.

The simplest instance of an epicyclic train is that of two wheels, one of which is carried around the other, mounted upon an arm which can rotate concentrically with the first wheel. This illustrates the basic principle of all epicyclic trains (Fig. 57). Power is applied to the arm A, and the wheel B is carried around C. Assume that the wheels are of equal size. If B is free to revolve upon its axis and C is fixed so that it cannot revolve, B will make two revolutions upon its own axis for each complete revolution of the arm around C. One of these revolutions will be due to the gear value or ratio of the train, and the other will be due to the movement of the arm. If the wheel B is one-half the diameter of C, it will make three revolutions about its own axis whilst the arm makes one revolution around C. Two of these revolutions will be due to the gear value of the train and the other will be due to the revolution made by the arm. The operation may be reversed, C being allowed to rotate about its own axis, and B constrained by some means so that it cannot rotate upon the stud or journal upon which it is mounted. That is to say, a mark upon it will remain pointing in the same

direction throughout the revolution of the arm. If the wheels are of equal diameters *C* will make two revolutions during one revolution of the arm. If the wheel *B* is one-half the diameter of *C*, the latter will make one and a half revolutions about its own axis whilst the arm makes one revolution. The half revolution will be due to the gear value of the train, the one revolution will be due to the revolution of the arm.

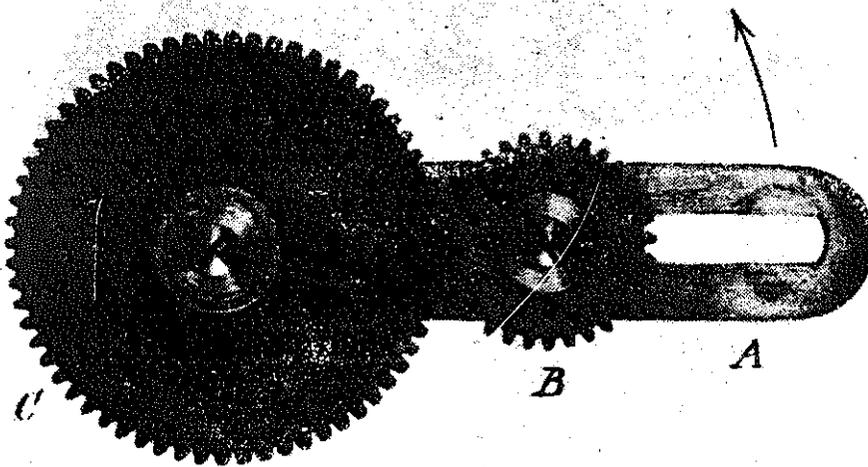


Fig. 57.—Epicyclic train of two wheels

Any number of wheels may be arranged along the arm, but unless some independent movement is given to one or other of them, those which are between the first and last wheels of the train act merely as idle or carrier wheels. For example, if a wheel *D* (Fig. 58), free to rotate upon its stud or journal, is interposed between *B* and *C*, the relative motions of *B* and *C* due to the epicyclic principle will not be affected. The only effect of intermediate wheels is upon the relative direction of rotation between *B* and *C*. But if some independent movement is given to *B* or *C*, or to one or more of the intermediate wheels, the rotations of the last wheel of the train, that is *B* or *C*, will be altered accordingly. In the general applications of epicyclic trains of gearing, the motion is derived only from the movement of the arm, but in all cases the fundamental principle is that

there will be one rotation of the last wheel, either plus or minus, due to the rotation of the arm in addition to that given by the gear ratio of the train.

With internal gears a disc is frequently used instead of an arm ; nevertheless it is still the arm, but in the guise of a disc. For example, A (Fig. 59) is a disc carrying wheels D in gear with wheels B and C, the former being an internally toothed wheel. Either B or C may be a fixed wheel ;  $D_1, D_2, D_3, D_4$ , so far as the relative motion between B and C is concerned, are merely idle or carrier wheels. The motion therefore will be one revolution, due to the rotation of the disc, plus the motion due to the gear value of the train. The four intermediate wheels, D, are used for mechanical reasons in order to distribute the wear, but this has no effect upon the epicyclic value of the train which

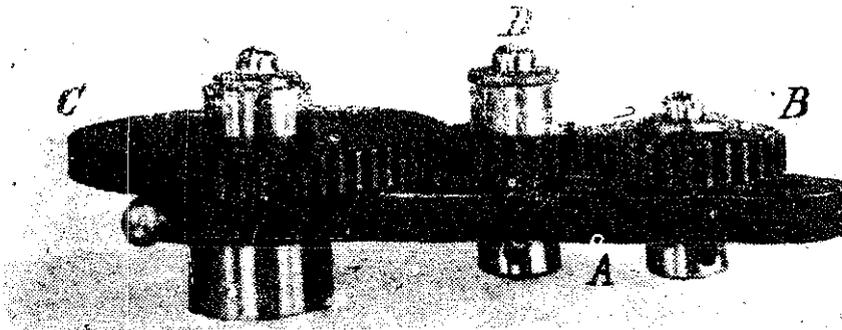


Fig. 58.—Epicyclic train of three wheels

really consists of any one of the intermediate wheels together with the centre and outside wheels, and the arm in the form of a disc. The epicyclic principle is applicable to trains of bevel gearing ; for example, three bevel gears, one of which is carried upon an arm attached to a central shaft and gearing with two others placed side by side. The wheel on the arm can rotate upon its own axis ; of the others, one is fixed and one is free to rotate upon the shaft. When the shaft is rotated the wheel which is free upon it receives a compound motion ; it will receive one revolution due to the motion of the arm, and also rotation due to the gear value of the train. As the wheels are of equal size,

it will receive two revolutions for each revolution of the shaft. The arrangement of two wheels in an epicyclic train is frequently termed a "sun and planet" gear.

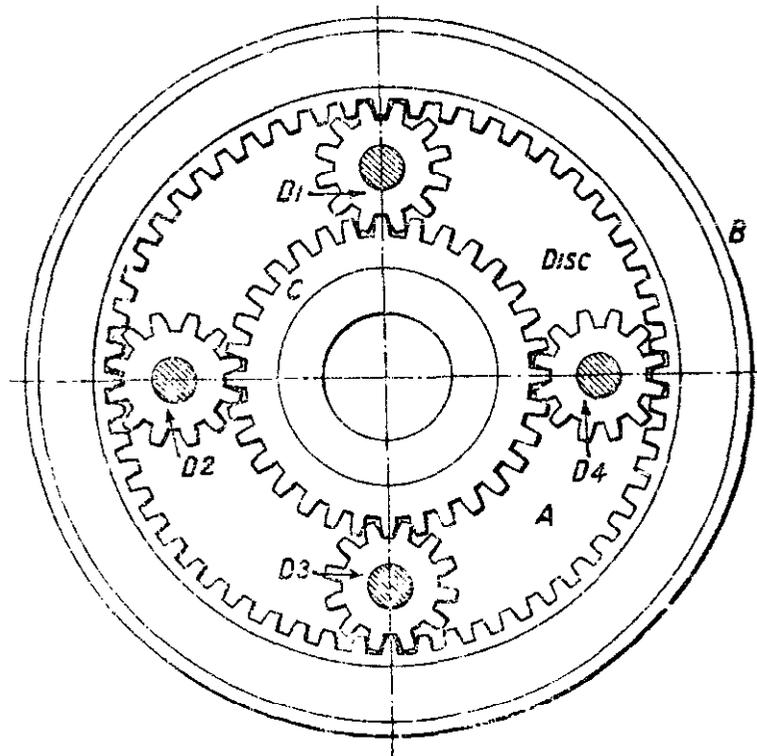


Fig. 59.—Internal epicyclic gearing

### The "Marlborough" Gear

There is an arrangement of gearing which should not be mistaken for an epicyclic train; it is used for gearing two parallel shafts when their axes are very close together; Fig. 60 shows an example. Shafts A and B are each provided with a gear wheel; a pinion or wheel C upon another shaft gears into both of these. The rotation of A is thus transmitted to B through the medium of C, which is merely an idle or carrier wheel. If, however, C were mounted upon an arm and the motion derived from the rotation of the arm around the other wheels, the arrangement would then be an epicyclic

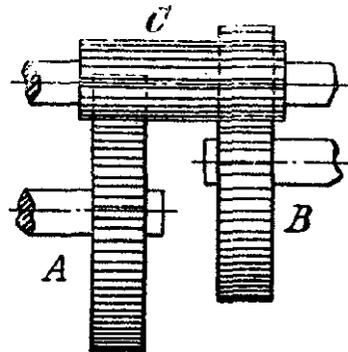


Fig. 60.—Marlborough wheel gearing

train, and the shafts A and B would have to be in the same axial line. Gear wheels are not always circular; there are elliptic, square, heart-shaped, lobed, and other forms, the object of their use being to obtain varying velocities between the driving and driven shafts.

### Knuckle Gearing

There is a type of gear wheel used where great strength is necessary for the teeth in order that they may stand heavy and rough working. The teeth are of simple form, being merely shaped to

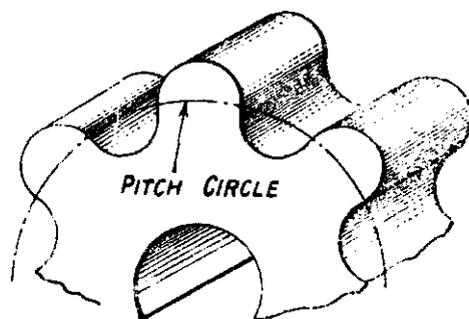


Fig. 61.—“Knuckle” teeth

arcs of circles, and are very short (Fig. 61). This type of gearing is only suitable for slow motion, and when noise and jar and an uneven velocity ratio between the shafts can be tolerated on account of the gain in strength.

It is a makeshift, wrong in theory, but usable for practical reasons

and because there is very little oblique thrust. Strictly it is equivalent, so far as the form and action of the teeth are concerned, to lantern or pin-wheel gearing. But, as explained in Chapter III, with gearing of this kind to be correct in principle, the pins should be on one wheel only, and that should be the driven wheel. With knuckle gearing there are really two pin wheels in gear with each other; the principle is incorrect. In practice, the teeth are sometimes modified in shape; but the more this departs from the simple circular form, the less will be the advantage of strength and simplicity for which knuckle teeth are adopted. Knuckle gearing is an example of construction, wrong in principle, yet of utility, applicable to and successful for purposes within its limitations.

## CHAPTER VII

### Definitions and Calculations for Cutting Involute Gears

BEFORE describing gear cutting, it will be as well to give a résumé of definitions, adding to these a few simple practical problems.

*Diameter* of a gear invariably refers to the pitch circle; the *Outside Diameter* is the size of the blank required.

*Diametral Pitch* is the number of teeth in each inch of pitch diameter.

For example, a gear with 48 teeth has a pitch diameter of 6 in. The diametral pitch is  $\frac{48}{6} = 8$ . The gear is thus 8 pitch.

If the outside diameter is known, and also the number of teeth; to get the diametral pitch, add 2 to the number of teeth and divide by the outside diameter. Thus: if the outside diameter is  $6\frac{1}{2}$  in. and there are 48 teeth;  $48 + 2 = 50$ , divide by  $6\frac{1}{2}$ , result, 8 pitch.

*Circular Pitch* is the distance along the circumference of the pitch circle between the centres of adjacent teeth.

If the diametral pitch is known, then circular pitch is found dividing this into 3.1416. Thus 8 pitch measures  $\frac{3.1416}{8} =$

0.3927 in. between adjacent teeth.

Conversely, diametral pitch is found by dividing 3.1416 by circular pitch.

*Module* is the pitch diameter in millimetres divided by the number of teeth, and the pitch diameter in millimetres is equal to the module  $\times$  number of teeth in the gear. The module can

be converted into inch diametral pitch by dividing it into 25.4. Thus Module 1 = 25.4 diametral pitch, 2 = 12.7, 3 = 8.46, 4 = 6.35 and so on.

*Pitch Diameter* can be found by dividing the number of teeth by the diametral pitch.

Thus : number of teeth 48, and diametral pitch 8 give  $\frac{48}{8} = 6$  in. pitch diameter.

*Outside Diameter*, equal to the diameter of the wheel blank, is found from the number of teeth and diametral pitch by adding 2 to the number of teeth and dividing by the pitch.

Thus : number of teeth  $48 + 2 = 50$  ; diametral pitch = 8, outside diameter =  $\frac{50}{8} = 6\frac{1}{4}$  in.

*Number of Teeth*, given pitch diameter and diametral pitch, is found by multiplying these two numbers together.

If the diametral pitch and the outside diameter are known, then multiply the two figures together and subtract 2, and the result is the number of teeth wanted.

*Distances between Centres of Gears*, knowing their diametral pitch and numbers of teeth, can be found by adding together the numbers of teeth, halving the sum and dividing this by the diametral pitch. Thus : two gears of 8 pitch have 48 and 16 teeth ; adding these  $48 + 16 = 64$  : halve this,  $\frac{64}{2} = 32$ , divide by pitch,  $\frac{32}{8} = 4$  in. between centres.

*Dimensions of Teeth*. Thickness at the pitch line is half the circular pitch, or 1.57 divided by the diametral pitch. Thus : for diametral pitch 8,  $\frac{1.57}{8} = 0.196$  in.

The whole depth of the tooth is  $0.686 \times$  circular pitch, or  $2.157 \div$  diametral pitch.

Thus : for diametral pitch 8,  $\frac{2.157}{8} = 0.269$  in. or circular pitch  $0.3927 \times 0.686 = 0.269$  in.

The convenience of the system of reckoning in terms of diametral pitch can be seen at once, for diametral pitches are standardised in whole numbers, and in fractions in the larger sizes; circular pitches can only be reckoned to several places of decimals. The reason for this is that the factor which governs the relation between the diameter and the circumference of a circle is not a whole number, nor is it an exact one, for though it has been taken to a hundred decimal places and more, yet it has never been precisely established.

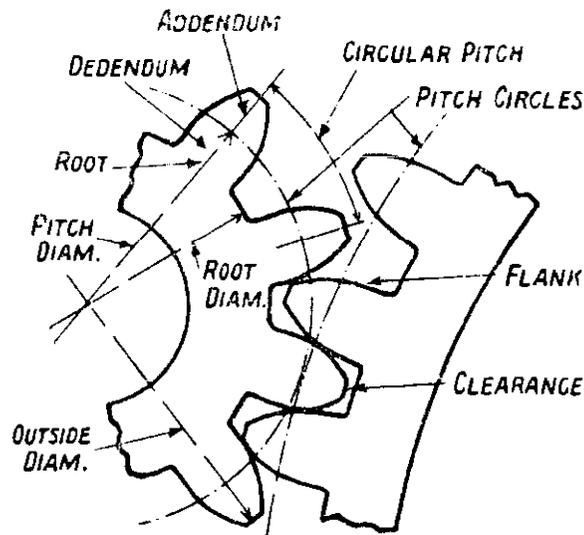


Fig. 62.—Measurements of gear wheel teeth

Sufficient exactness, however, is obtained by using standardised diametral pitch. The makers of milling cutters provide a range of cutters of from 1 to 48 pitch, and as it is useful to know the variety of circular pitches these provide, these pitches as well as the tooth depth are shown in the table on the following page.

All gears of any one given diametral pitch will mate together satisfactorily so long as they have been cut correctly. When cutting gears by the milling process, if a variety of sizes have to be produced, several cutters will be needed for each diametral pitch. The reason for this is shown in the diagram, Fig. 63, which illustrates a portion of a 10-tooth pinion and part of a 36-tooth wheel. Very little observation is required to note the difference in profile of the tooth spaces which are formed by the milling cutter. For this reason, cutter makers standardise eight shapes of cutter for each diametral pitch and, in addition, a series of half sizes where greater accuracy is needed. The wheels that the standard cutters are designed to cut are also shown overleaf.

## GEAR WHEELS AND GEAR CUTTING

## DIMENSIONS GIVEN BY STANDARD CUTTER PITCHES

Diametral Pitch	Circular Pitch	Depth of Tooth
1	3.1416	2.157
1 $\frac{1}{2}$	2.5133	1.726
1 $\frac{3}{4}$	2.0944	1.438
1 $\frac{1}{4}$	1.7952	1.233
2	1.571	1.079
2 $\frac{1}{4}$	1.396	0.959
2 $\frac{1}{2}$	1.257	0.863
2 $\frac{3}{4}$	1.142	0.784
3	1.047	0.719
3 $\frac{1}{2}$	0.898	0.616
4	0.785	0.539
5	0.628	0.431
6	0.524	0.360
7	0.449	0.308
8	0.393	0.270
9	0.349	0.240
10	0.314	0.216
11	0.286	0.196
12	0.262	0.180
14	0.224	0.154
16	0.196	0.135
18	0.175	0.120
20	0.157	0.108
22	0.143	0.098
24	0.131	0.090
26	0.121	0.083
28	0.112	0.077
30	0.105	0.072
32	0.098	0.067
36	0.087	0.060
40	0.079	0.054
48	0.065	0.045

## WHEEL SIZES FOR STANDARD CUTTER

No. 8 cuts gears of 12 to 13 teeth  
 No. 7 cuts gears of 14 to 16 teeth  
 No. 6 cuts gears of 17 to 20 teeth  
 No. 5 cuts gears of 21 to 25 teeth  
 No. 4 cuts gears of 26 to 34 teeth  
 No. 3 cuts gears of 35 to 54 teeth  
 No. 2 cuts gears of 55 to 134 teeth  
 No. 1 cuts gears of 134 to a rack

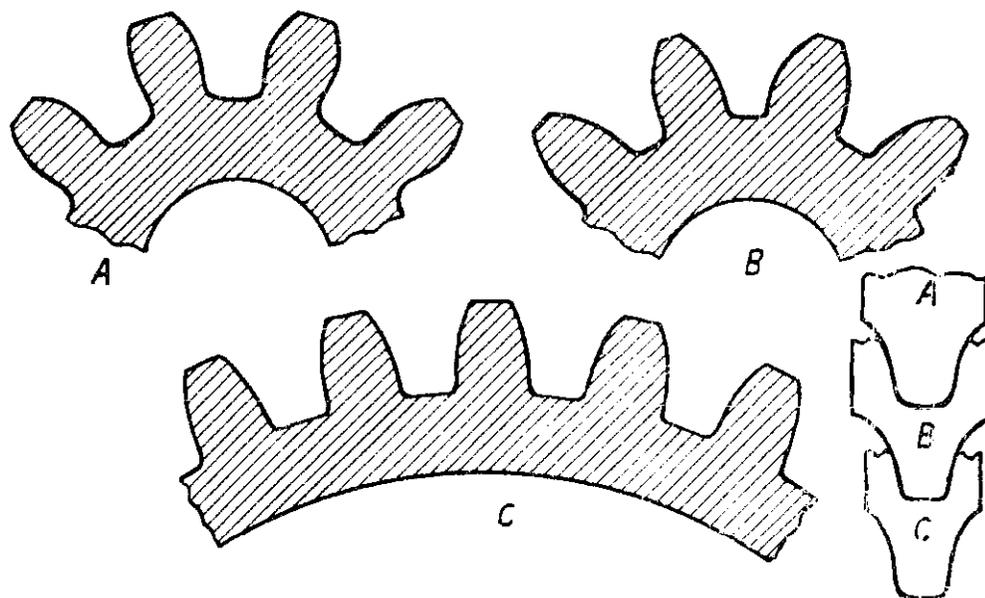


Fig. 63.—Varied shapes of cutters for differing diametrical pitches

### Simple Gearing Problems

#### *Velocity Ratio of Wheels in Gear*

R.P.M. of driver of 27 teeth is 1,800, at what speed will it turn the driven wheel that has 108 teeth ?

$$\begin{aligned} \text{The equation is : } D \times R &= d \times r \\ \text{which becomes } 27 \times 1,800 &= 108 \times r \\ \text{so } r &= \frac{27 \times 1,800}{108} = 450 \text{ r.p.m.} \end{aligned}$$

In a compound train the driver wheels have 27, 36 and 48 teeth and the r.p.m. of the first is 1,800. The driven wheels have 108, 120 and 180 teeth ; what is the speed of the final shaft ?

$$\begin{aligned} \text{As before : } D \times D \times D \times R &= d \times d \times d \times r \\ \text{which comes to } 27 \times 36 \times 48 \times 1,800 &= 108 \times 120 \times 180 \times r \\ \text{so } r &= \frac{27 \times 36 \times 48 \times 1,800}{108 \times 120 \times 180} = 36 \text{ r.p.m.} \end{aligned}$$

The layshaft in a gear box has to run at 1,150 r.p.m. ( $v$ ) and the driving shaft at 2,000 r.p.m. ( $V$ ). If they are at 10 inch centres ( $D$ ), what must be the diameters of the connecting gears ?

Let  $R$  and  $r$  be the radii of the gears : then

$$r = \frac{v \times D}{V + v} = \frac{1,150}{3,150} + 10 = 3.65 \text{ in.}$$

$$R = D - r = 6.35 \text{ in.}$$

Diameters required are 7.3 and 12.7 in.

To achieve the object set out in the above problem, design a suitable pair of gear wheels.

Trying first 8 diametral pitch :

$$\text{Number of teeth in both gears} = 2 \times D \times P = 20 \times 8 = 160.$$

$$\begin{aligned} \text{Number of teeth in small gear (n)} &= \frac{160 \times V}{V + v} = \frac{160 \times 1,150}{3,150} \\ &= 58 \end{aligned}$$

$$\text{So number of teeth in large gear} = 160 - 58 = 102.$$

$$\text{Speed of layshaft is } \frac{58}{102} \times 2,000 = 1,137 \text{ r.p.m.}$$

Trying 10 pitch next :

$$N + n = 2 \times D \times P = 20 \times 10 = 200$$

$$n = \frac{200 \times V}{V + v} = \frac{200 \times 1,150}{3,150} = 73 \text{ teeth}$$

$$N = 200 - 73 = 127.$$

$$\text{Speed of layshaft is } \frac{73}{127} \times 2,000 = 1,149 \text{ r.p.m.}$$

This is as near as can be expected, so the terms of the problem are solved by the use of gears of 10 pitch, having 73 and 127 teeth respectively. As will be shown later, these are rather awkward numbers, but they serve as examples of how the calculation is made.

As sprocket wheels for chain driving are frequently employed in machine work, a problem dealing with these may be added. To calculate the diameter of a roller chain sprocket, Chordal Pitch is used. Chordal pitch is measured in a straight line, along the pitch circle, between the centres of adjacent teeth, that is, the pitch of the chain.

If  $N$  = number of teeth in sprocket

$P$  = pitch of chain

$d$  = chain roller diameter

As shown in Fig. 63a,

$$\text{Angle } A = \frac{360^\circ}{2N} \text{ and the pitch diameter} = \frac{P}{\sin A}$$

Outside Diameter will be Pitch Diameter + D.

Diameter at bottom will be Pitch Diameter - D.

Find the pitch diameter of a sprocket having 18 teeth, chain  $\frac{1}{2}$  inch pitch, roller diameter 0.30 in.

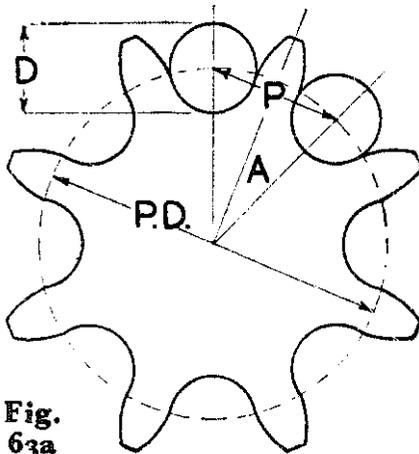


Fig.  
63a

$$A = \frac{360}{2 \times 18} = 10^\circ$$

$$\text{and P.D.} = \frac{0.500}{\sin 10^\circ} = \frac{0.500}{0.1736} = 2.88 \text{ in.}$$

$$\text{Outside diameter} = 2.88 + 0.30 = 3.18 \text{ in.}$$

$$\text{Bottom diameter} = 2.88 - 0.30 = 2.58 \text{ in.}$$

### Facts in Brief

The weakest point in a train of gearing is the smallest pinion in that train.

The wider the rim of the wheel, the stronger are the teeth. A gear with teeth of fine pitch and a broad rim is stronger than one with teeth of coarse pitch and a narrow rim.

The velocity ratio between the first and last wheels in a simple train is not affected by the insertion of intermediate wheels. These wheels do however affect the direction of rotation of the first and last wheels, but not the relative number of revolutions made by the driving and driven shafts. An odd number of intermediate wheels preserves the direction of rotation; an even number reverses it.

In a compound train of wheels, the first and last wheels revolve in the same direction. To reverse the relative motion of these wheels a single intermediate wheel may be inserted anywhere in the train without affecting the velocity ratio of the first and last wheels. This is useful when cutting left-hand threads in a screw-cutting lathe.

## CHAPTER VIII

### Gear Cutting in Milling Machines

FROM the foregoing chapters in this book enough will have been learned to show that a properly designed gear is a matter of precision, and that exact methods of manufacture are essential.

There are, speaking generally, two main methods of making gear wheels in use today. The enormous demands of the motor industry for gears that will run silently, and will be strong enough to withstand the stresses in the motor-car gearbox, have produced a large variety of gear cutting machines. Such gears as these are invariably cut from solid blanks, and this cutting may be carried out in one of two different ways : direct cutting by a milling cutter, or generating by hob or planer.

Die-casting is a process also used for producing small gears ; here liquid metal is forced into a mould, under considerable pressure, resulting in a clean profile that needs no machining. Gears of this sort are relatively expensive, and only a very large production figure would justify the expense of the dies. Another method of producing small gear wheels or pinions for clocks, watches, and scientific instruments, where a single wheel or a very small number is required, is by the use of a fly-cutter. This is a single-pointed cutter, accurately shaped to the space between the teeth, and driven at a high speed by suitably arranged belting. Wheels can be cut in an ordinary lathe by this process, in conjunction with a dividing head.

#### Gear Cutting by Milling

The time-honoured method of gear cutting, still much used, is by employing a milling machine, using specially formed cutters,

and a dividing head to move the blank round step by step at the correct intervals. A sketch (Fig. 64) shows the general arrangement. The blank is mounted upon an arbor, adequately supported against the pressure of the cutter and carried at one end by the

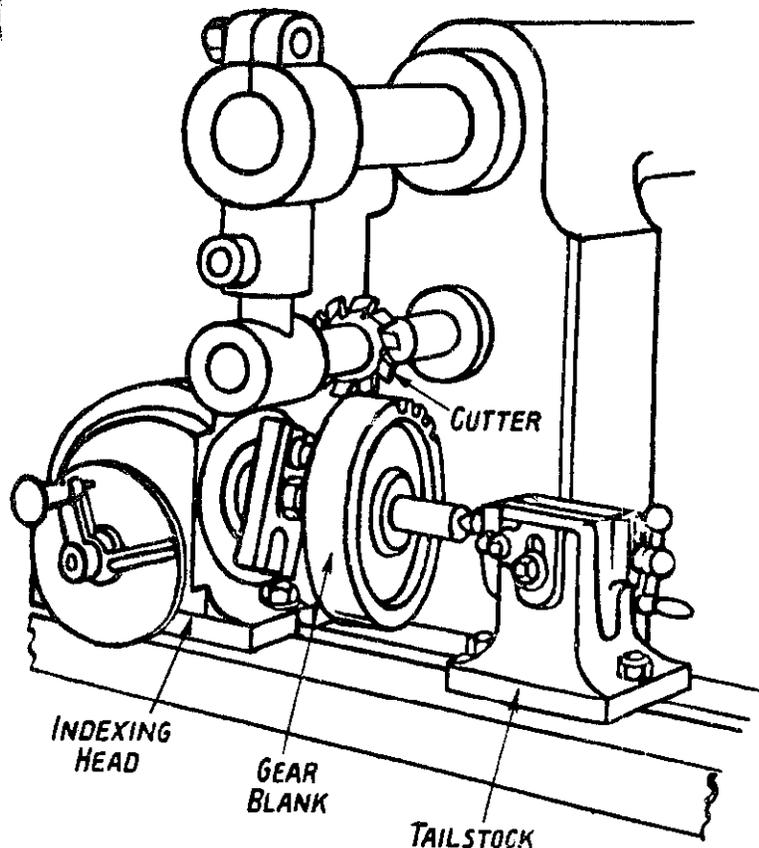


Fig. 64.—Gear cutting in a milling machine

headstock, with its dividing mechanism, and at the other end by a tailstock centre. An adjustable strut can be brought into contact with the rim of the wheel to counteract the cutting pressure.

The feed is provided in the usual way by the table screw, actuated by suitable change gears. In some quarters, where cutting down of production time is important, the feed is increased to the maximum, short of heating the cutter unduly, but instead of proceeding immediately to the next tooth space, the blank is rotated by several spaces. This gives the part of the blank heated by the machining a chance to cool down.

Fixed directly to the spindle of the dividing head is a disc having 24 notches around its rim, and by the use of this any number of divisions can be made that are factors of 24, that is : 2, 3, 4, 6, 8, 12. This is the plain dividing head.

### Universal Head

The universal head is fitted with a number of steel discs, in each of which a number of circles of accurately spaced holes has been drilled, usually six. A typical set will include plates having 15, 16, 17, 18, 19 and 20 holes ; 21, 23, 27, 29, 31 and 33 holes ; and 37, 39, 41, 43, 47 and 49 holes.

The spindle of the universal head is driven by a worm and worm wheel, with a ratio of 40 to 1, through a crank and handle. Thus, one turn of the crank advances the spindle  $1/40$  of a turn, so by this means a gear of any number of teeth that will divide exactly into 40 can be accurately spaced out : that is to say, 5, 8, 10, 20 and 40 teeth, by making 8, 5, 4, 2 or 1 turns of the crank respectively. The idea of the index plate can now be seen, for it enables the operator to make exact part-turns of the crank. A peg that locks the crank handle in any position can be pushed into any of the holes in the plate (see Fig. 65).

When using this plate the rule is : divide 40 (the worm and wheel ratio) by the number of spaces to be set out in order to obtain the number of turns or parts of a turn to be given to the crank handle. Consider a gear of 28 teeth. Dividing 40 by 28 we get  $1$  and  $12/28$  of a turn, but there is no index circle with 28 holes, so some other similar fraction must be worked out, thus :—

$$\frac{12}{28} = \frac{3}{7} = \frac{21}{49}, \text{ and as we have a 49 hole circle, each adjustment is}$$

made with one whole turn and 21 holes in addition.

Another example may show the idea more fully. The number of divisions wanted is, say, 60. The division shows that each new cut must be made after  $40/60$  of a turn has been given to the crank. But there is no disc with a 60 hole circle, so we must

find one with a circle divisible by 3, such as 39. The calculation is as follows :

$$\frac{40}{60} = \frac{2}{3} = \frac{26}{39}, \text{ so each movement is made}$$

by advancing the crank handle 26 holes.

To facilitate spacing out the teeth, a pair of arms is pivoted upon the crank spindle, and these can be clamped at any angle relative to each other, so that they span an exact number of holes. Each forward step is made within the space included between the two arms.

With 18 circles of holes a very wide range of divisions can be made, but it is by no means complete. Every division up to 50 can be made, but thenceforward to 100 and beyond, there are

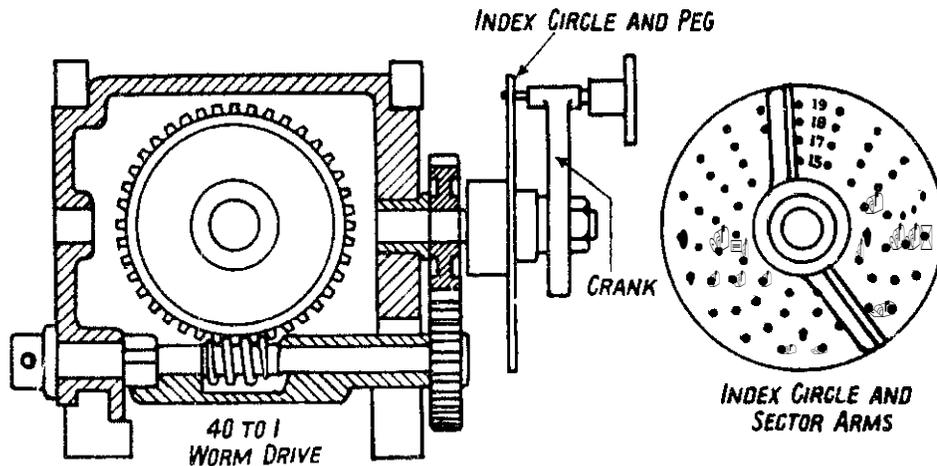


Fig. 65.—Diagram of universal head and division plate

many gaps that can only be covered by the system known as differential indexing. Here, the crank handle is geared to the worm spindle, but the method is a little complicated, and its description must be left to a more advanced book than this.

The example given at the end of the last chapter, in which two gears were designed, resulted in wheels having 73 and 127 teeth. Both of these would need differential indexing, as practically all prime numbers—numbers that have no factors—are unobtainable by direct indexing.

In milling gear teeth it is possible, of course, to clamp several blanks together, thus producing more than one wheel at a time.

This depends upon the capacity of the machine. But if the need is for a considerable production of a variety of gears, then the use of a gear-cutting machine is to be recommended.

### Bevel Gear Cutting

The more difficult process of bevel gear cutting on a plain milling machine is one that must be mentioned, even though that machine is not the ideal one for this purpose. A little consideration of the form of these gears will show why this is so. The blank for a bevel gear is a part of a cone, consequently both the teeth and the spaces are tapering in shape. As a result of this, no cutter of rotary type can be devised to mill out this space in a single operation, and even when the necessary cuts have been made, the teeth will have to be finished by hand to get an exact profile.

The tooth space is narrower at one end than at the other. The cutter cannot be wider than the narrower end, and if it is made to give the correct profile there, then the profile will not be correct at the larger end. Two cuts have to be taken, with the gear blank set off from the true centre, first to one side and then to the other ; the blank also has to be rotated in order to give the correct width at the larger end (see Fig. 66 and Fig. 67). The cutter will give the proper profile at this end, and so, if bevel gears have to be produced by milling, some hand-finishing will be essential.

The amount of the set-over can be calculated with the aid of a table of constants. These constants are based upon the width of the tooth faces compared with the whole length of the cone side of which the surface is a part.

The selection of cutters is not a straightforward matter, as a different cutter will be used for each wheel, that is, of course, if they are of different sizes. Cutter makers publish tables for this purpose, but these are too extensive to insert here. Further information on the generation of bevel gears is given in the next chapter.

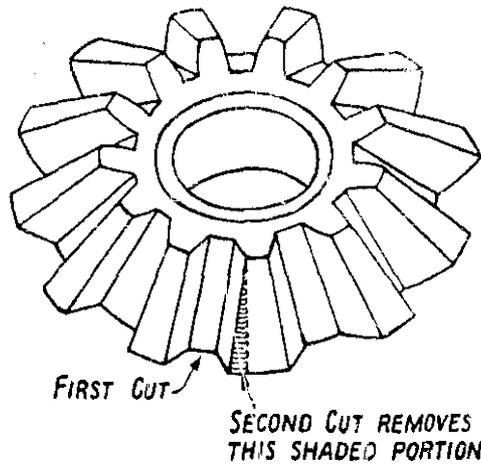
Laying Out Blanks

It is as well that the reader should know how to lay out the shape of blanks for a bevel

Fig. 66.-Milling the teeth of a bevel wheel

gear and pinion ; this is by no means as simple a matter as for the spur gear blank. Assume that a pinion of 32 teeth is to drive a gear of 72 teeth, 8 diametral pitch. The process is shown in Fig. 68.

First draw two centre lines intersecting at right angles, PP for the pinion, GG for the gear. On either side of these draw lines QQ and RR for the pinion, spaced apart as many eighths of an inch as there



Lines through these intersections, at right angles to the diagonals, represent the backs, and  $\frac{1}{8}$  in. on each side of these indicate the top and bottom of the teeth. The width of the tooth faces is a matter that depends upon the load they are to carry and the speed at which the wheels are to run. In the same way the thickness of the webs and the length of the shaft bosses also depend upon the work the wheels have to do.

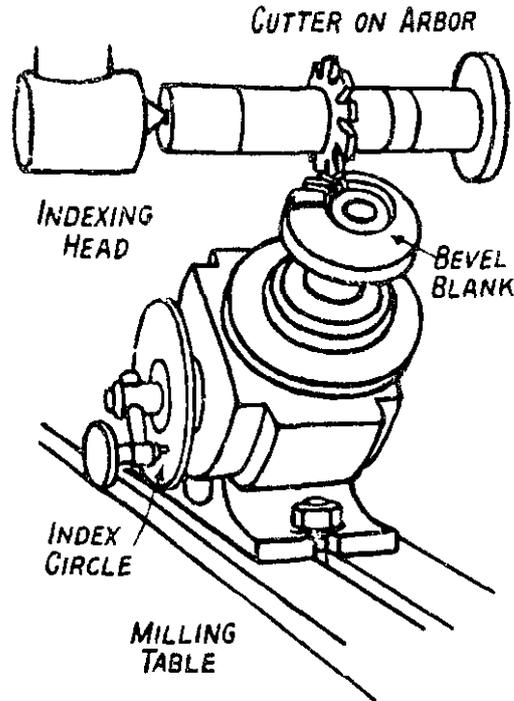


Fig. 67.—The two cuts in milling bevel-wheel teeth

are teeth in the pinion—32 in this case—and HH and JJ for the gear, 72 eighths apart and parallel to the centre lines. Join up to the centre C the diagonals CD, these represent the pitch lines.

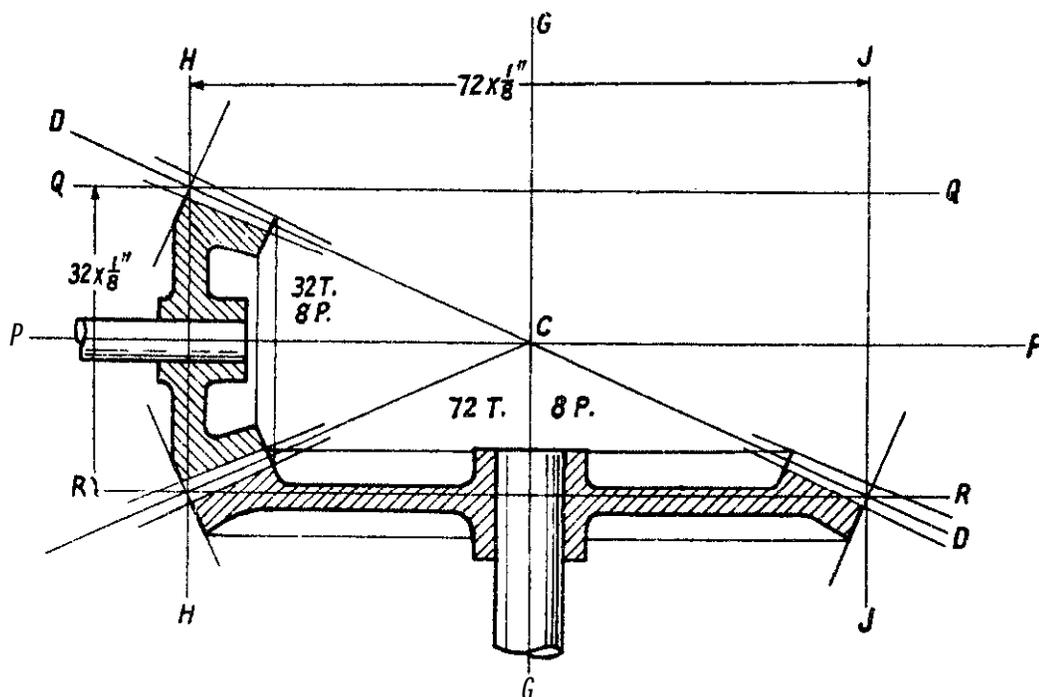


Fig. 68.—Laying out blanks for bevel wheel and pinion

Gear wheel blanks in cast iron or gun-metal may be cut dry. Steel blanks should be cut with a plentiful supply of a suitable cutting lubricant.

It is important that the bore of a gear blank should be perfectly concentric with the circumference of the blank, to ensure even meshing of the teeth and smoothness of working. The bore should also be of the correct size to fit accurately on to the mandrel or other fixture carrying the blank in the machine.

Where silent working is especially desired, pinions with machine-cut teeth of rawhide are sometimes used. They work smoothly and quietly, require little or no lubrication, and are approximately equal in strength to cast iron gears. These pinions can be used to mesh with gears cut in any metal. The layers of rawhide are clamped together by metal discs on either side of the pinion to prevent spreading of the hide through the lateral strain incurred in working.

Modern practice, however, tends to favour wheels made from fabric impregnated with synthetic resins. This material provides the necessary silence in operation; in addition it has good mechanical strength and does not readily deform.

## CHAPTER IX

### Cutting Spur Gears in the Lathe

IN Chapter VIII it was pointed out that gear wheels can be cut in the lathe. This may be done in two ways. In Method 1 the gear blank is supported in a fixture attached to the lathe cross slide and the cutter is rotated by the lathe mandrel. In Method 2 the exact opposite procedure is adopted, the work being held in the chuck or mounted between centres on an arbor and the cutter is revolved in an attachment bolted to the top slide. If gears of heavy section are to be cut, the first method is preferable as there is usually greater inherent rigidity with this arrangement, to say nothing of the increased driving power available. The second method is, however, very satisfactory for cutting the lighter type of gear and it has the merit that the operator's view of the work is generally less restricted.

#### Dividing Devices

In both the above methods some form of dividing attachment is required and this may be either simple or elaborate as the work demands. The simplest form of dividing device is a master gear attached to the spindle carrying the gear blank. A detent to engage the tooth spaces of the master gear is provided and the gear is rotated progressively tooth by tooth until all teeth in the blank have been cut. This will give an exact replica of the master gear or will enable gears to be cut that are multiples of the master gear. Fig. 69 shows a simple arrangement of this type. The principle of using a master gear can be applied to either of the aforementioned methods of gear cutting ; in the

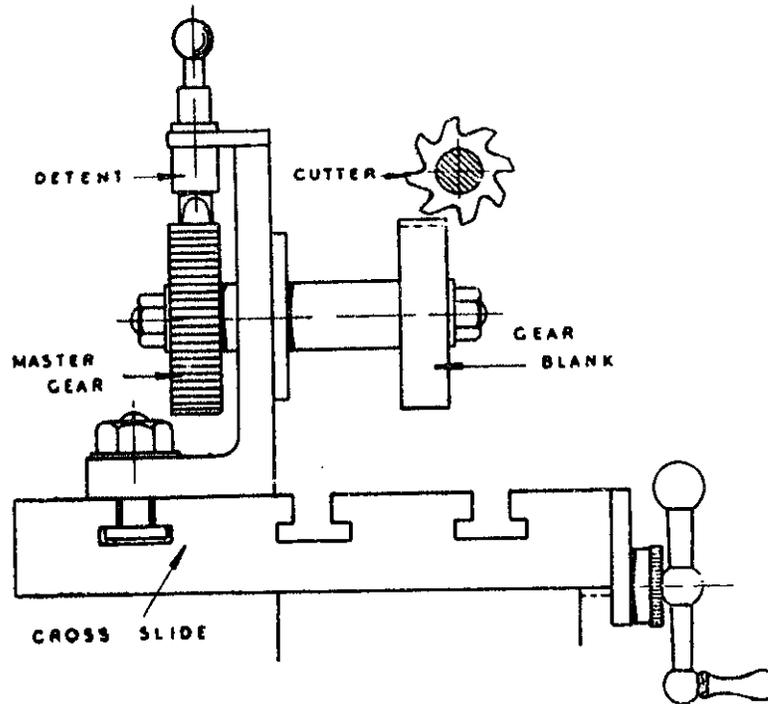


Fig. 69.—Simple indexing head on cross-slide of lathe

first method the device can be used as shown, and in the second the master gear can be mounted on a suitable adapter, forming an extension to the lathe mandrel. The gear blank is then held in the chuck or mounted on an arbor. In this case, the detent to engage the teeth of the master wheel is carried in an extension of the change wheel quadrant, as shown in Fig. 70.

A more elaborate device for dividing is that shown in Fig. 71. This is a combination fixture and is intended to be bolted to a vertical milling slide attached to the lathe cross-slide. It does, in fact, form a complete milling attachment, as it is provided with a worm

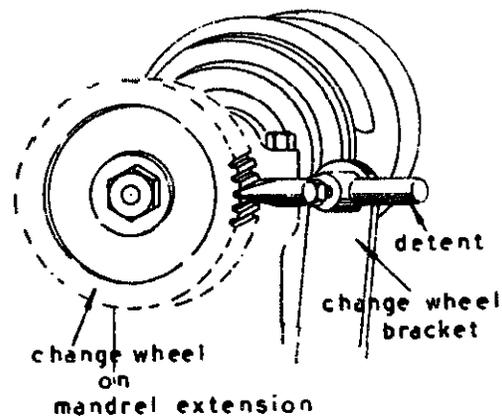


Fig. 70.—Indexing by change gear on lathe mandrel

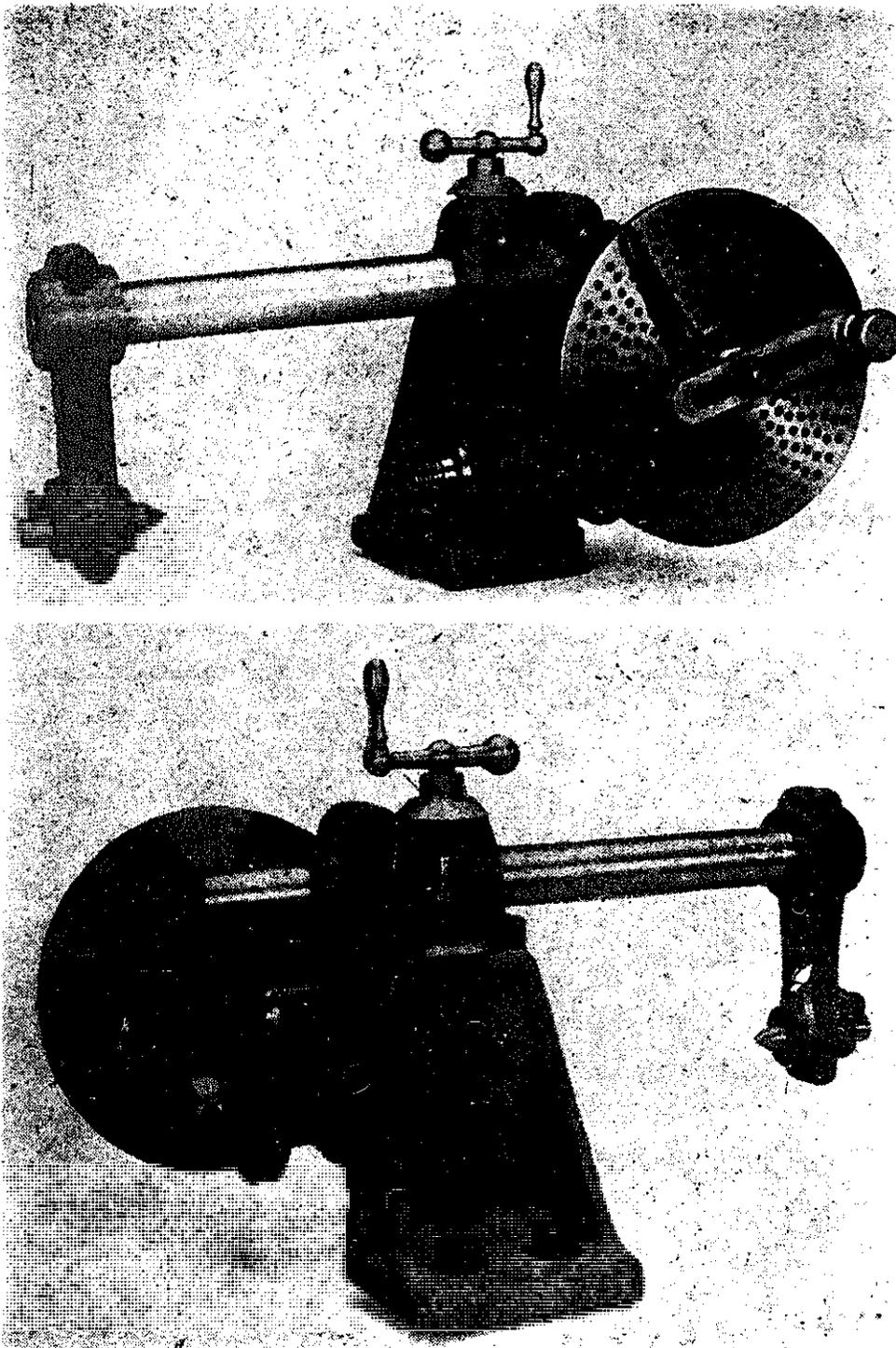
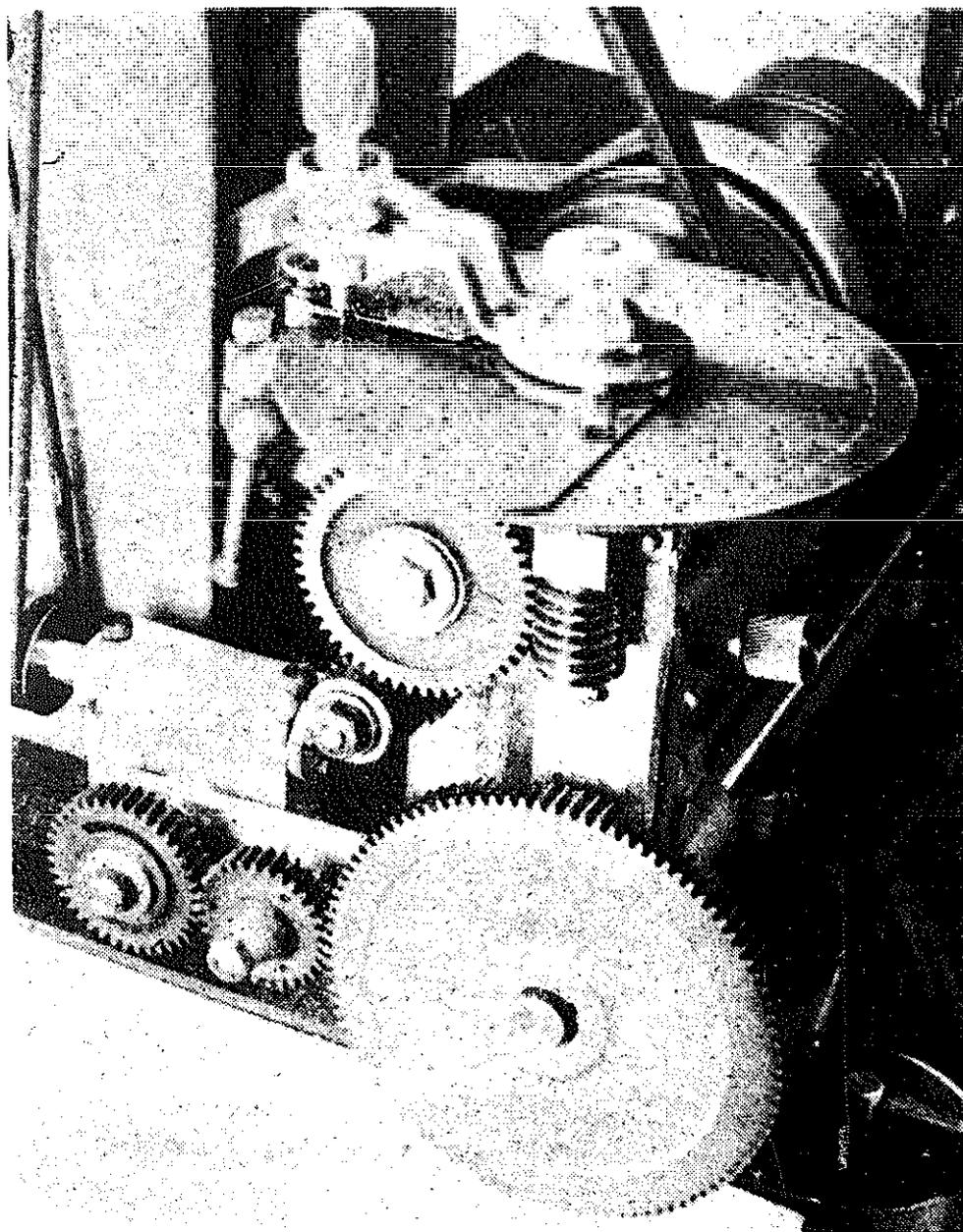


Fig 71.—Two views of the Myford dividing attachment to fit cross-slide of lathe

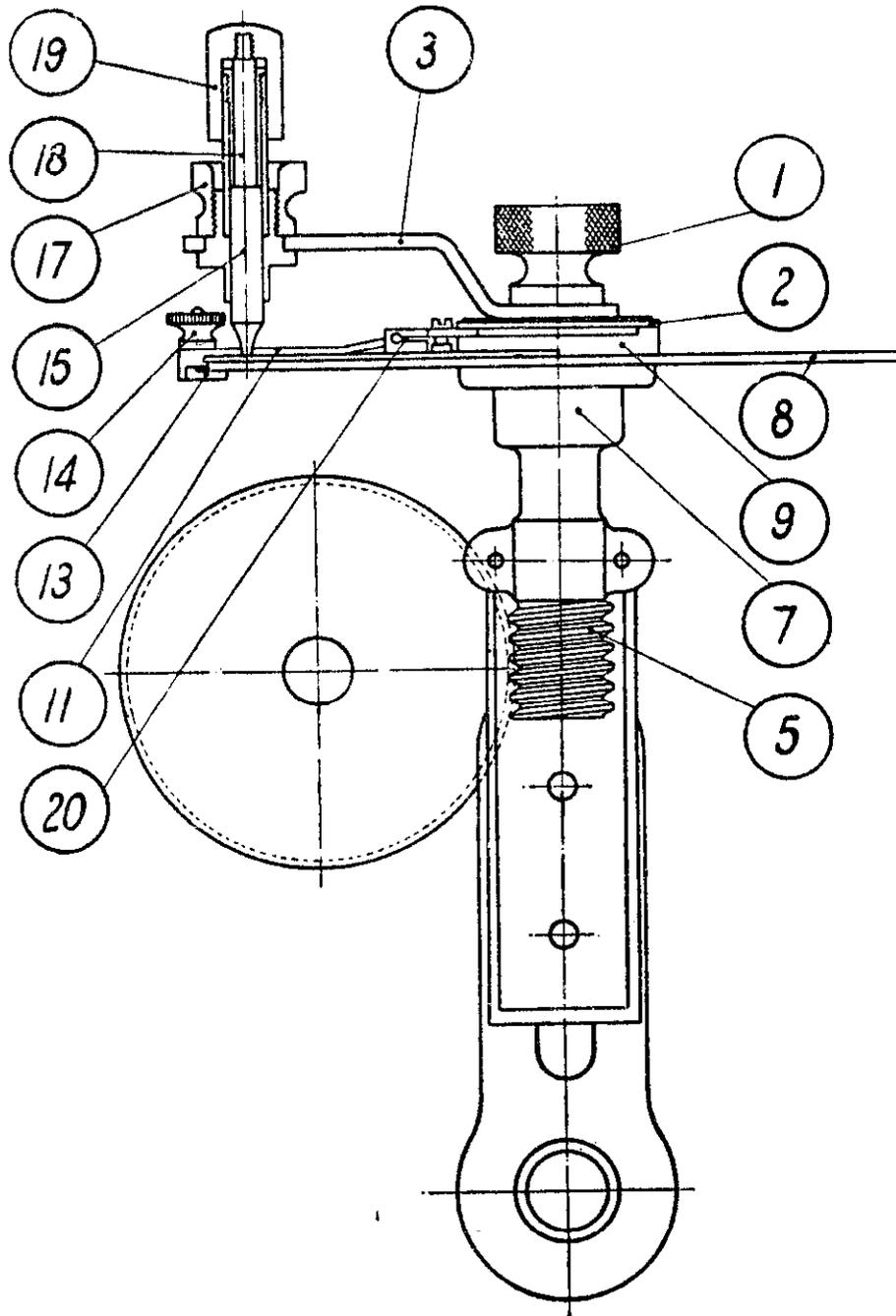


**Fig. 72.—Worm-gear dividing appliance on lathe mandrel**

dividing head, as described in Chapter VIII, and also a means of holding the gear blanks.

A worm Dividing Head may also be applied to the lathe mandrel and this is shown set up in Fig. 72 with its worm engaging a standard change wheel mounted on the mandrel extension.

With such an arrangement it is possible to put any of the lathe change wheels on to the mandrel extension and to engage them



73.—Details of worm dividing appliance

with the worm of the dividing head. This greatly facilitates dividing and avoids the use of a multiplicity of the division plates described in the previous chapter.

As will be seen in Figs. 73, 74, and 75, such a dividing head is of relatively simple construction and is well within the capabilities of the amateur machinist. This attachment is carried on

a mounting that can be bolted to the change-wheel bracket, thus making its adjustment for mesh with the change wheel a simple and rapid process.

With regard to the mandrel extension or change wheel adapter, the form shown in Fig. 76 has proved very reliable and is suitable for any lathe having a mandrel bore of  $\frac{5}{8}$  in. and upwards. Below this size, it may not be possible to accommodate this form of adapter, in which case it will be necessary to thread the bore accurately and to screw in a stub extension to take the change wheel. It must be emphasised that accuracy is essential in this matter as eccentric running of the wheels cannot be tolerated.

It will be appreciated that, with a little ingenuity it is possible to drive the leadscrew of the lathe independently and so to

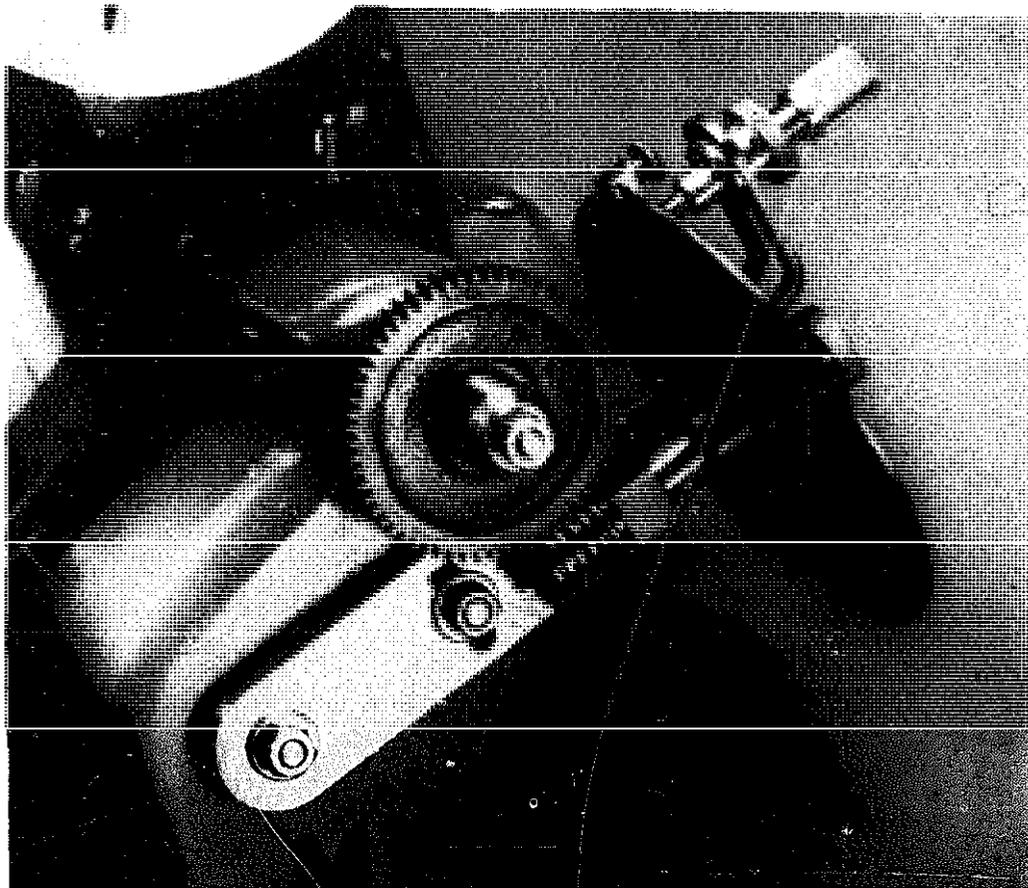


Fig. 74.—The dividing head mounted on the ML7 lathe

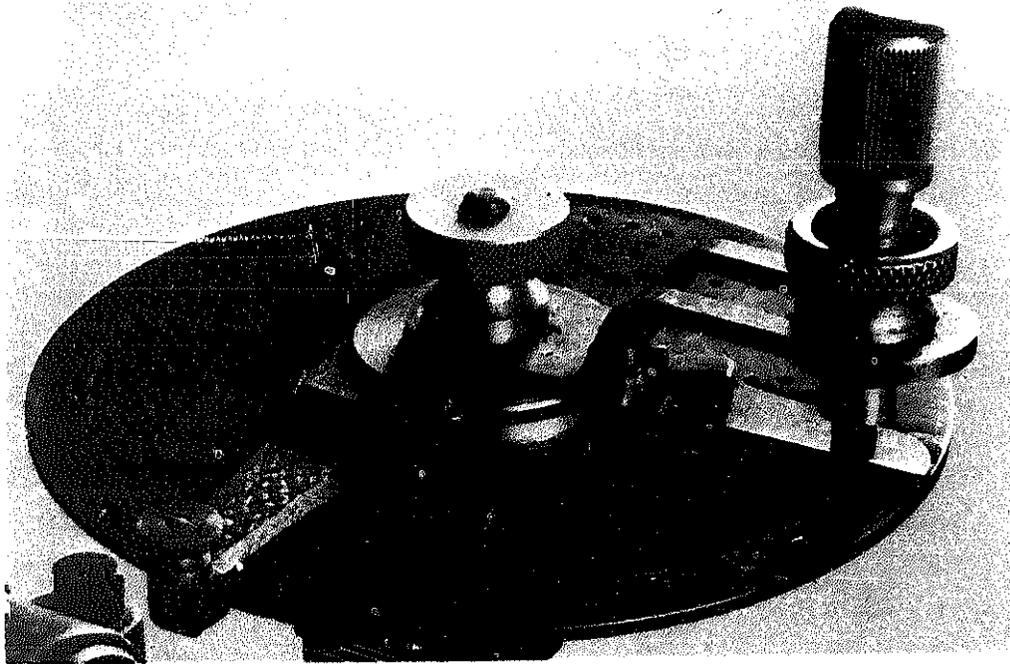


Fig. 75.—Division-plate and sectors

provide an automatic feed when cutting gears by the second method; that is when the cutter is rotated in an attachment bolted to the saddle and independently driven. When cutting by the first method, this form of drive can be used provided the cross-slide has power feed.

In order to design such an arrangement, it is necessary to work out the correct rate of feed for the cutter, having regard to its diameter and the material being machined. It is assumed that the peripheral speed of the cutter will have been correctly adjusted to suit the material, in accordance with the methods which will be described later.

As will be seen in Fig. 72, in addition to the change wheel bracket, there is a further quadrant mounted on the end of the lathe carrying a reduction gear box and a train of gears to drive the leadscrew of the lathe. The pulley on the gearbox is driven from the lathe countershaft from which the belt to the lathe mandrel has been unshipped as it is not required.

### Calculating Correct Feeds

The correct train of gears to drive the leadscrew is arrived at in the following way : the amount of metal removed by a cutter of  $1\frac{1}{8}$  in. diameter (this being a convenient size for gear cutting in the small lathe) in one revolution varies from about 0.001 in. for tool steel to some 0.030 in. for aluminium. Assume the r.p.m. of the cutter at the lowest driven speed is 60 r.p.m. and the highest speed 200 r.p.m. Therefore, when cutting at 0.001 in. per revolution, the cutter will remove 0.060 in. per minute at the low speed and 0.240 in. per minute at the high ; that is, the amount removed per minute is the amount removed per revolution multiplied by the r.p.m. of the cutter.

Now suppose the feed screw has a  $\frac{1}{8}$  in. lead, then one turn of

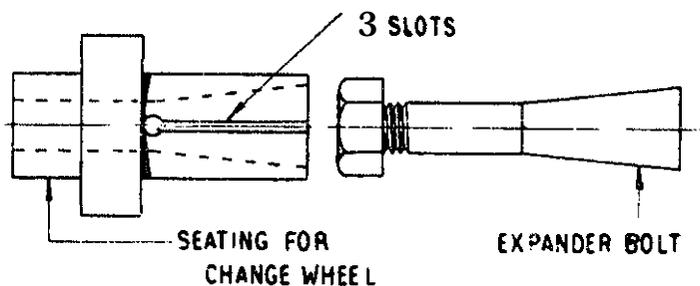


Fig. 76.—Expanding plug mandrel to carry change gears

the feed screw per minute will give  $\frac{1}{8}$  in. or 0.125 in. of feed per minute. So to feed 0.060 in. per minute, the lead screw must

revolve  $\frac{60}{125}$  turns per minute or .48 of a turn, say half a turn.

Similarly if it is required to feed at 0.240 in. per minute the lead screw must revolve at  $\frac{240}{125}$  turns per minute that is 1.925

turns, say 2 turns.

The worm reduction gear, which is driven from the lathe countershaft, has a reduction of eighty to one and the lead screw is driven from this through a train of gears for final adjustment of the rate of feed. Let us suppose that the lathe countershaft rotates normally at 400 r.p.m., then the worm wheel shaft

of the reduction gear will make 5 turns per minute. To make the leadscrew revolve at two turns a minute, a train of gears, after the worm reduction gear, having a ratio of 5:2 will be correct and two gears of 100 T and 40 T can be used. Similarly, if half-a-turn per minute is required, the gear train must be in the ratio of 5 : 0.5. Unfortunately, no standard set of lathe change wheels will satisfy this, so the speed of the worm wheel shaft must be reduced to 200 r.p.m. by changing the ratio of the cone pulley on the countershaft, from which the reduction box is driven, to 2 : 1 from the original 1 : 1 ratio. This will leave a 2 . 5 : 0.5 reduction between the feed screw and the worm reduction box and this can be met by using two wheels of 100 T and 20 T.

To reduce the calculations to a formula, the number of turns the leadscrew must make is equal to the feed per minute required, divided by the pitch of the leadscrew, i.e.

$$\text{Number of turns equals } \frac{\text{Feed per minute required}}{\text{Pitch of lead screw}}$$

both feed and pitch being expressed in inches, and the ratio of reduction will be :

$$\frac{\text{Turns per minute of feed screw}}{\text{R.p.m. of countershaft}}$$

from which ratio the necessary train of gears can be set up.

### Calculating Correct Cutter Speed

To revert now to the calculation of correct peripheral cutter speed, to which reference has already been made.

Correct cutting speeds are a matter of finding from the chart on page 80 the speed in feet per minute at which the cutter should run, having regard to the rigidity of the gear cutting attachment and the nature of the workpiece. The speed of the cutter is then adjusted to the figure found and this may be reduced to the formula :—

$$\text{R.p.m. equals } \frac{\text{Feet per minute}}{0.295}$$

The figure 0.295 is a constant, depending on the diameter of

the cutter used (in this case  $1\frac{1}{8}$  in.), and is given by the formula

$$\frac{\pi \times d}{12} \text{ equals the number of feet travelled in one turn of the cutter, where } \pi \text{ is } 3.142 \text{ and } d \text{ is the diameter of the cutter in inches.}$$

Conversely, if the r.p.m. are known, the number of feet per minute the cutter shall run can be found from :

$$\frac{\pi \times d}{12} \times \text{r.p.m. equals feet per minute.}$$

or  $0.295 \times \text{r.p.m. equals feet per minute.}$

Material	Peripheral speed of curter ft. per min.
Soft grey iron . . . . .	70
Machine casting Cast-i& . . . . .	50-60
Cast steel . . . . .	40
Wrought iron . . . . .	45
Malleable iron . . . . .	40-45
Soft machine steel . . . . .	45
Hard " " . . . . .	30
Tool steel, annealed . . . . .	30-35
" " unannealed . . . . .	20-25
Soft brass . . . . .	125
Hard " . . . . .	90-100
Bronze" . . . . .	80
Aluminium . . . . .	400-600

Cutters are mounted on arbors, carried between centres, in the first method and on an accurately made adapter fitting the milling spindle in the second method. Details of this adapter and the method of mounting the cutter are shown in the frontispiece of this book. It will be seen that, in order to impart greater rigidity, a steady bearing, similar to the overarm bearing of the milling machine proper, is fitted. The desirability of ensuring that the cutter runs absolutely true cannot be too strongly emphasised, for a wobbling cutter will at once destroy any accuracy in the machined gear teeth.

### Driving the Cutters

When the first method of gear cutting in the lathe is employed, the cutter is, of course, driven by the lathe mandrel in the usual

way, but in the second method an independent drive is required. This may take the form of an elaborate overhead gear, allowing for automatic adjustment of the belt tension, like that shown in Figs. 77 and 78.

### Mounting and Centring the Work

Whichever method of cutting is used it is of the utmost importance to see that the gear blanks are secure on their mounting, whether it is a mandrel running between centres or an arbor held in the chuck. If there is any slackness in the mounting, the blank may move under the pressure of the cut and the tooth spacing will then be upset. Similarly, the mandrel

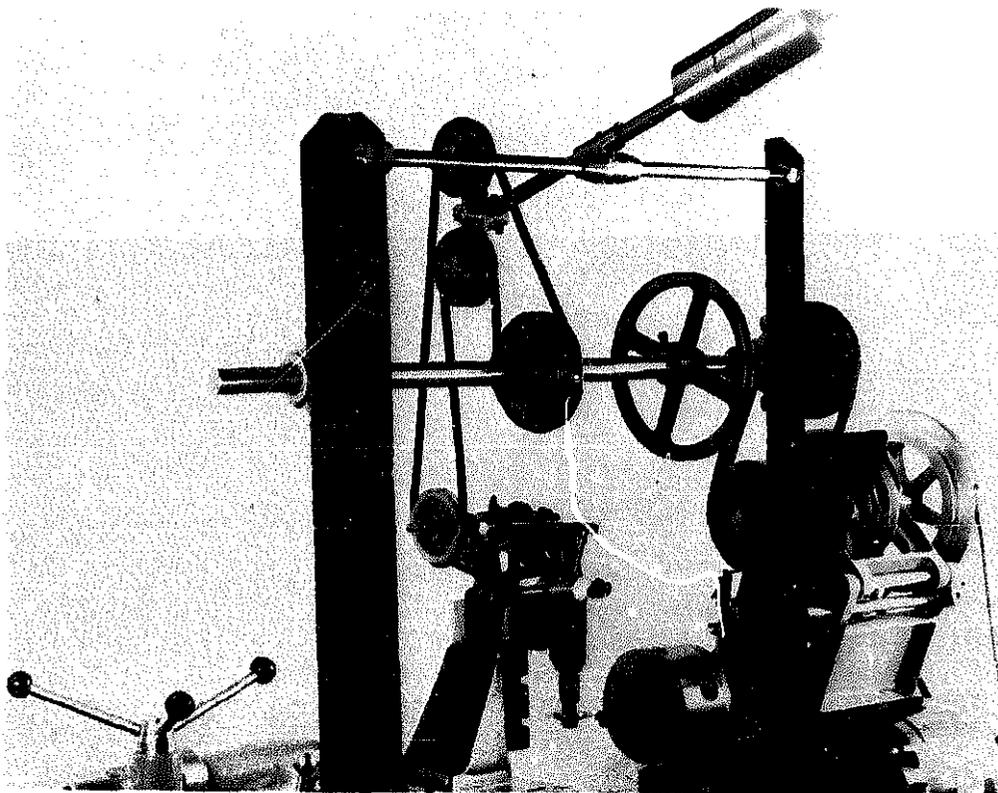


Fig. 77.—Overhead for gear cutting

must run perfectly true or a gear form eccentric to the blank diameter will be cut.

Of equal importance is the need to ensure that the centre of the cutter falls on the centre line of the gear blank ; an exaggerated error in this respect is shown in Fig. 79. A fairly simple method of ensuring this is shown in Fig. 80. This shows a blank in position on the mandrel ready for cutting and the gear

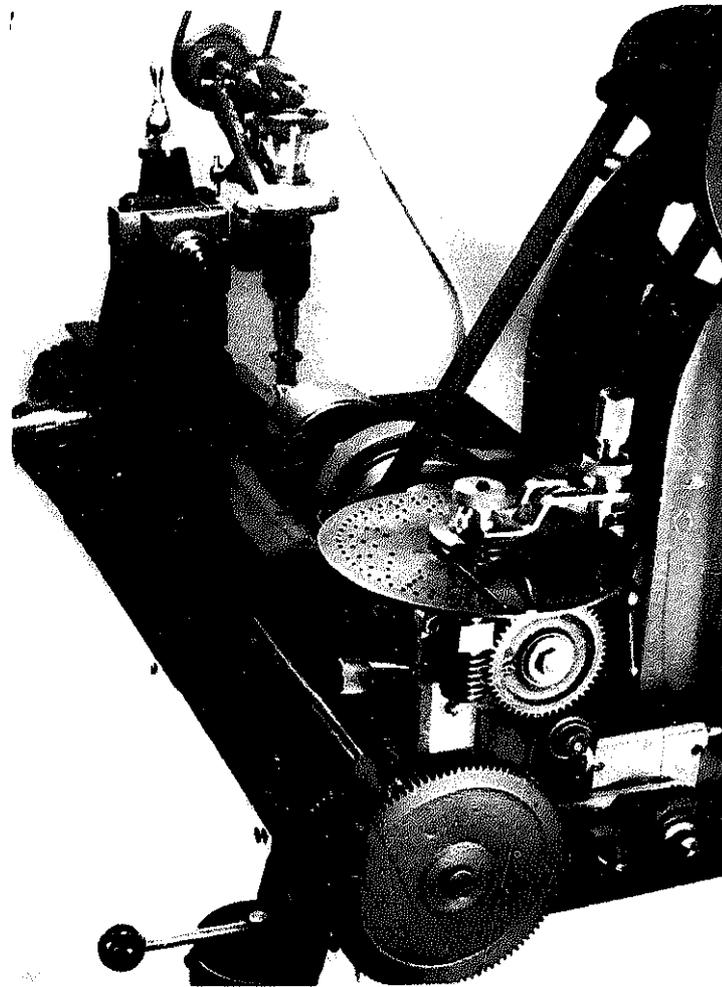


Fig. @.-Set-up for milling gears in lathe

cutter on its arbor ready for centring, the set-up being as for method 2. It is necessary to measure accurately the lettered dimensions shown in Fig. 80 so that an estimation of the measurement "e" can be obtained. When this has been arrived at, it is quite easy to set the cutter to this measurement from the lathe

bed by means of inside callipers to which a micrometer measurement has been transferred.

The measurement "e" equals  $C - \frac{d}{2}$

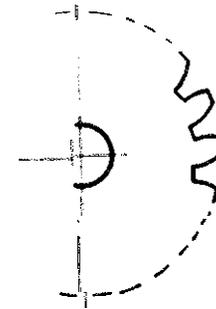
where "c" equals  $A + \frac{b}{2}$

and a, b, c, d are the measurements indicated in Fig. 80.

An alternative method of centring is to use a simple height gauge, having a blade provided with a V-notch which may be set to centre height from the lathe tailstock centre. This method is open to objection as, with fine diametral pitches, the accuracy of the setting may be in doubt owing to difficulty in determining exactly when the cutter is touching both sides of the V-notch.

The first method of centring is also applicable when the cutter is rotated by the lathe mandrel, all measurements shown as being taken from the lathe bed are then taken from the face of the chuck or from the driver plate, according to the way in which the cutter is mounted.

Fig. 79.  
Result of  
inaccurate  
centring  
of cutter



### Cutting The Teeth

In Chapter VII we have seen that in a spur gear the whole depth of tooth is given by the formula :—

$$\frac{2.157}{DP} \text{ equals whole depth of tooth.}$$

This figure is required for setting the depth of cut and, whether the operation is carried out by Method 1 or Method 2, the cut must be set to this depth so that the gear tooth can be fully formed at a single pass. This holds good for cutting small gears in the lathe, though for larger gears the practice is different. In any case the slide controlling the depth of cut must be set to zero. This setting is carried out preferably by inserting a

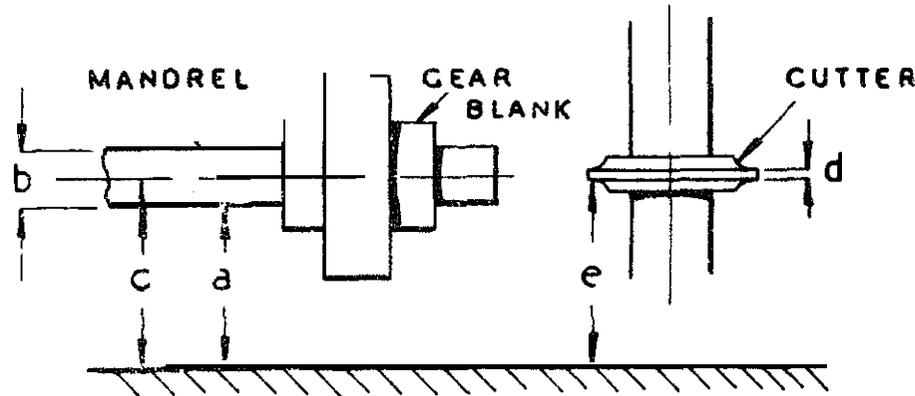


Fig. 80.—Method of centring cutter with gear blank feeler gauge between the work and the top of one of the cutter teeth, and the slide is then advanced by an amount equal to the thickness of the feeler gauge. At this stage, the slide index is at zero and the correct depth of cut is then put on. After making sure that the dividing device is also set at zero, the actual cutting may be started. When one tooth space is completed the cutter should be returned ready for the next cut, the dividing device is moved round for the next tooth, the cut is then taken and the whole procedure repeated until all the teeth are machined.

Generally, cutting should be carried out at as high a speed as the inherent rigidity of the machine and its attachments will allow.

As the illustration, Fig. 80A (on the back cover) will show, it is not impossible to cut an internal ring gear in the lathe. To do so the work itself is mounted in the 4-jaw chuck so that the flank from which the gear is to be cut can be set to turn truly. The operation is one of shaping each tooth separately by means of a form tool set in the lathe toolpost and traversed by its lathe saddle which is moved by hand against a simple stop. The work is, of course, indexed by the methods described earlier, while the whole depth of cut, obtained from the formula on the previous page, is set progressively by means of the cross slide index, cuts of 0.002 in. to 0.003 in. at a time being taken. Again a simple stop is set with reference to readings taken from the cross-slide index showing when the full depth of cut has been attained.

## CHAPTER X

### Gear Generating Methods

It is generally admitted that, if accurate gear teeth are wanted, the best system of producing them is by one of the generating methods. There are various processes in which the form of the tooth is generated by a rack tooth cutter, the wheel blank being rotated at the same time as the cutting is done. As every involute gear is able to mesh truly with an involute rack, and the rack tooth, as we have seen, has easily-formed straight sides, the method is excellent in principle.

#### Hobbing, Planing and Shaping

There are three methods of gear generating in use—hobbing, planing, shaping, and to a certain extent, grinding. The hob is actually a milling cutter in the form of a worm with a single thread, having straight sides making an included angle of 29 deg. Nicks are cut across to form cutting faces and the

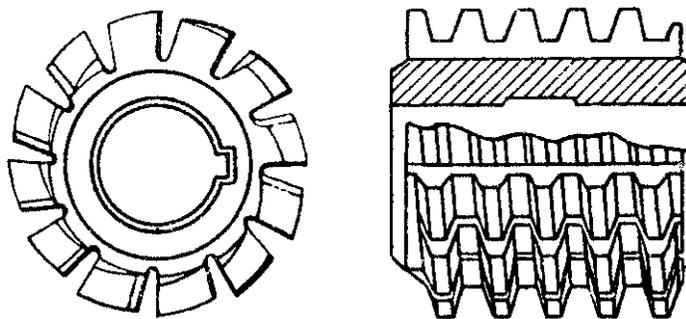


Fig. 81.—End and sectional views of hob

worm is relieved a little behind these faces to give cutting clearance. These hobs can be used in a milling machine or in specially designed hobbing machines. Fig. 81 shows such a hob.

As the hob has a worm thread, the blank will have to be set

over to ensure that the cut teeth will be truly parallel to the axis when ordinary spur gears have to be cut. Once cutting has

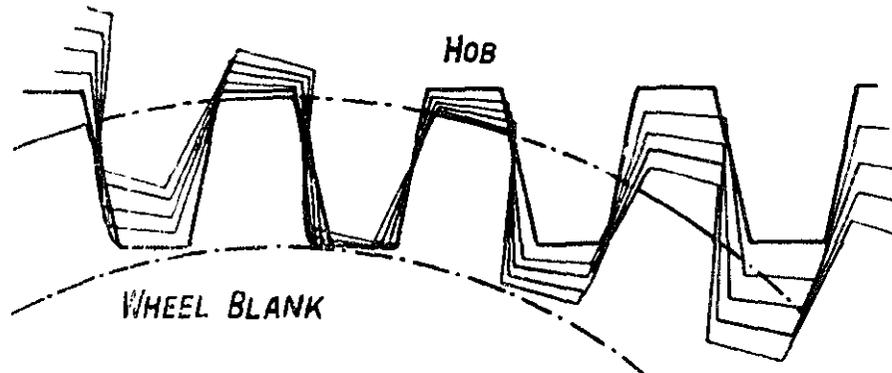


Fig. 82.—Generation of teeth by hob

commenced, the hob rotates the blank just as a worm rotates its worm wheel, and the teeth are thus truly formed in the manner shown in the drawing (Fig. 82).

This method can produce as nearly perfect a tooth form as the condition of the cutting tool and the machineability of the metal

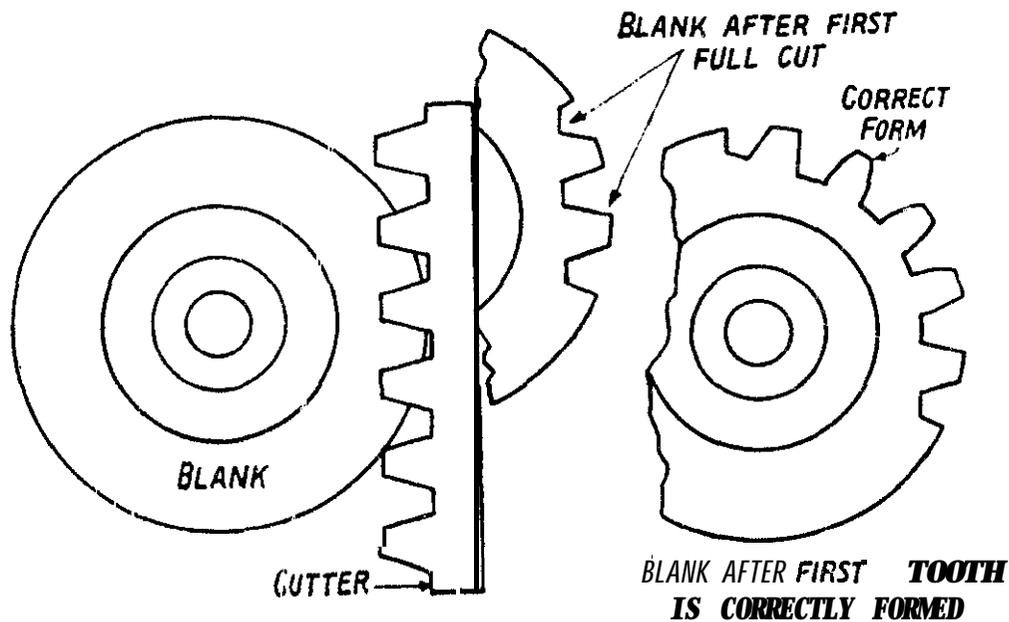


Fig. 83.—Principle of Sunderland gear planer

will permit. It suggests the moulding of teeth in a plastic material blank, rolled in contact with a set of teeth of perfect form. Fortunately, this latter condition is easy to attain because it exists in the straight-sided rack tooth. The plastic wheel blank

is perhaps not yet available, but it is possible to use a planing cutter formed with rack teeth and to plane out the spaces in a rotating blank with it.

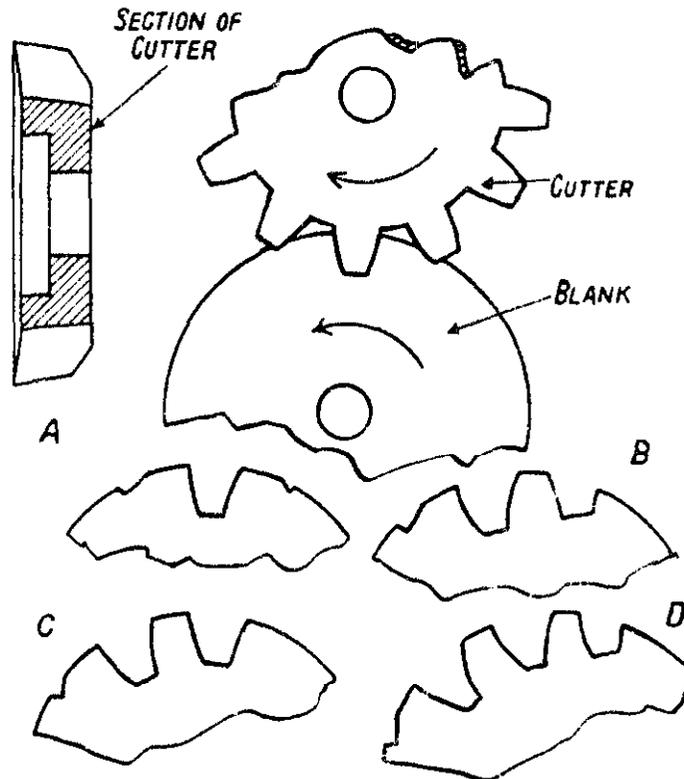


Fig. 84.—Principle of Fellows gear shaper

This is the principle of the Sunderland gear planer, shown diagrammatically in Fig. 83. On the left is seen, in plan view, the cutter after having made a full depth cut with the blank held stationary. This stage completed, the blank is rotated and the result is to produce a truly formed tooth in much the same way as does the hob. On the right are shown two stages of the blank, removed from contact with the cutter. The cutter is moved to and fro to make successive strokes, as well as up or down in gear with the rotating blank.

A similar idea is applied in the Fellows gear shaper, but here the cutter is in the form of a complete gear that rotates with the wheel blank after a preliminary cut has been made. This also is shown (see Fig. 84) with four sections of the blank, A, B, C, D, indicating progressive stages in machining. The left-hand flanks

of the teeth are correctly formed first, and the right side is not completed until the teeth come into position after making the first revolution.

Bevel wheels are best produced upon a planer—a machine with a somewhat complex action, in which two cutters each machine one side of a tooth. Their motion is controlled by moulded templates to give the correct contour, and a device brings them gradually closer together, so that they converge upon the apex of one wheel cone, as they are given a forward motion on the cutting stroke. There are several such machines on the market, but as they are highly specialised in their action they do not come within the scope of this book.

### Helical Gears

Another important cutting process is the production of the helical, or so-called “ spiral ” gears. These are not difficult to make, and the work can be done either in a lathe or in a milling machine, the latter for preference. When the nature of the teeth of a helical gear is understood, as it should be after reading the section devoted to it, it becomes clear that the wheels are simply sections cut off a bar on which a multi-start thread has been turned. If the operator can turn such a thread as this he can equally well produce helical gears. The number of starts is equal to the number of teeth in the wheel, and this means that a dividing head will have to be used, though the competent lathe hand knows how to mark the teeth on the change wheels to produce similar results.

In milling helical gears, the blank has to be rotated at the same time as it is carried forward under the cutter, and the degree of rotation must be strictly governed. To produce this result, change wheels have to be used just as in a lathe. The calculation is simple. The lead of the helix (or spiral) and the lead of the table screw are the two factors. The formula is as follows :—

$$\frac{\text{Lead of required helix}}{\text{Lead of machine table}} = \frac{\text{Driven gears}}{\text{Driving gears}}$$

Lead screws of milling machines and lathes usually have four

threads to the inch, and the index head a 40 to 1 worm drive. If the two are geared together by equal wheels, then one turn of the lead screw will produce 1/40 turn of the index head spindle and, by the time a complete turn of the latter has been made, the table will have been moved 10 in. The lead of the table is 10 in.

Four change wheels have to be considered, one on the lead

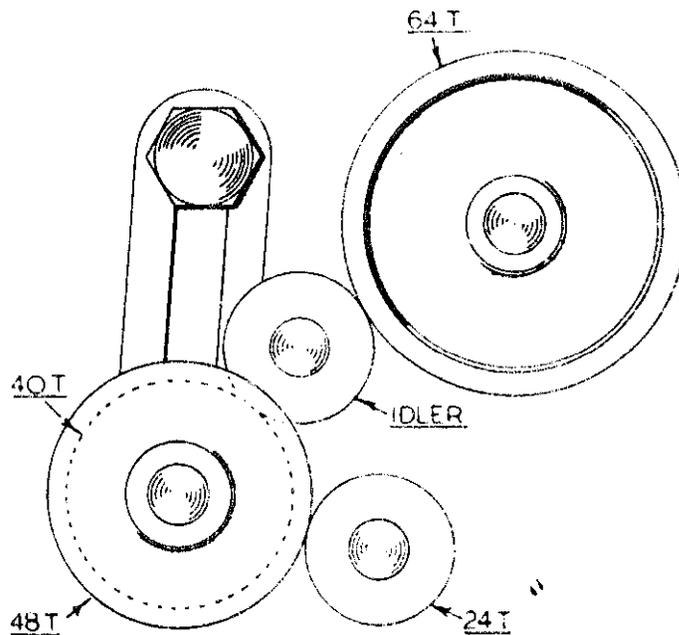


Fig. 85.—Use of idler wheel for left-hand helix

screw, one on the worm, and the two intermediate wheels on the studs. An idler wheel will be needed if the helix is left-handed and this is in centre as the diagram Fig. 85 shows. The usual equipment of change wheels for milling machines is two of 24 and one each of 28, 32, 40, 44, 48, 56, 64, 72, 86 and 100 teeth respectively.

As an example, consider a gear with a helix of 32 in. lead. From the formula just given :—

$$\frac{\text{Lead of helix}}{\text{Lead of table}} = \frac{32}{10}, \text{ which can be expressed as}$$

$$\frac{4 \times 8}{2 \times 5}, \text{ or as } \frac{4 \times 12}{2 \times 12} \times \frac{8}{5} \times \frac{8}{8} = \frac{48}{24} \times \frac{64}{40}$$

Out of these, 24 and 40 will be the drivers, and 48 and 64 the followers arranged as in the diagram Fig. 86 : 24t. on the lead

screw, 64t. on the worm spindle, 40t. the first and 48t. the second.

As another example, consider a helix of 6 in. to be cut on the same machine. Proceeding as before

$$\begin{array}{r} \text{Lead of helix} \\ \text{Lead of table} \end{array} \quad \begin{array}{r} 6 \\ 10 \end{array} = \frac{2 \times 3}{5 \times 2} = \frac{40 \times 48}{100 \times 32}$$

So the drivers are 100 and 32 and the followers 40 and 48.

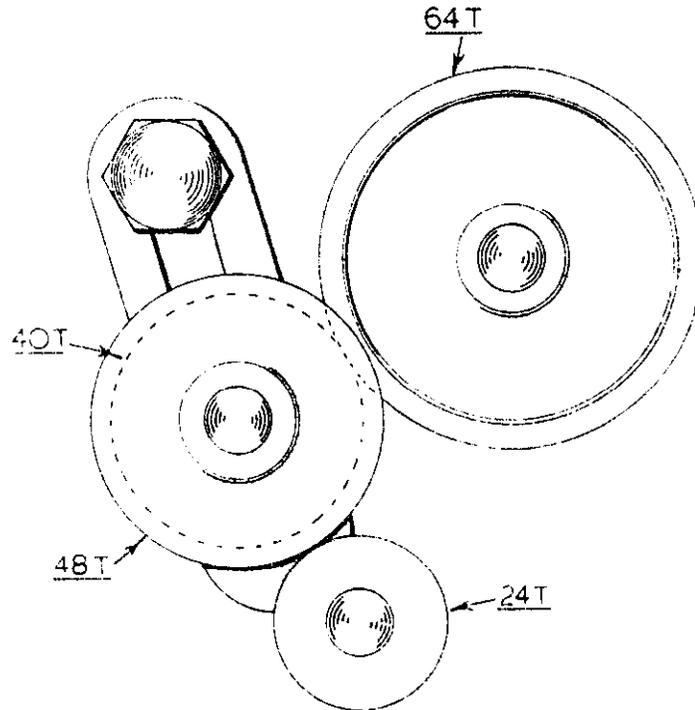
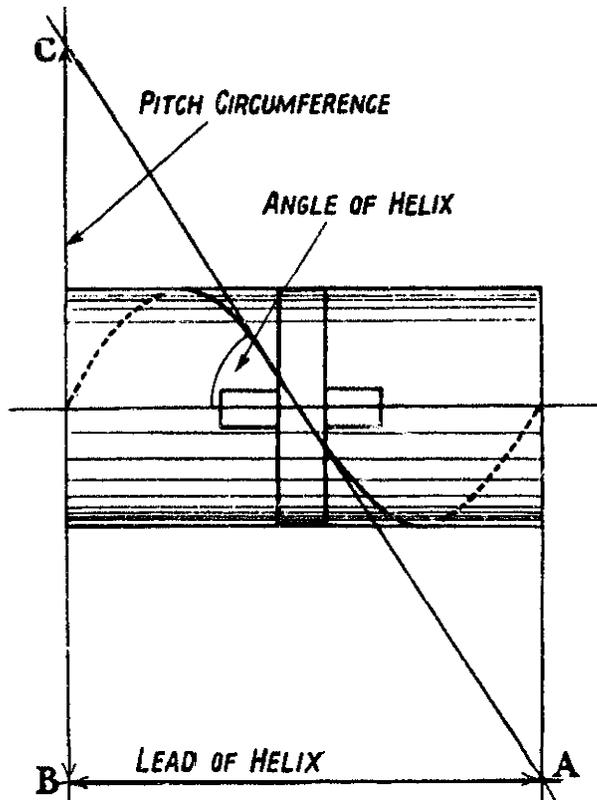


Fig. 86.—Arrangement for milling helical gears

In addition to giving this motion to the blank, the table will have to be set over at an angle equal to the helix angle, a term that needs explanation. In the diagram (Fig. 87) a blank wheel is shown, with one tooth line drawn across it and projected on either side to show the true course of the helix. The pitch or lead of the curve is equal to the distance marked AB, and the helix angle is that at which the straight line AC crosses the wheel. This line indicates the angle made by the helix with a line at right angles to the wheel axis at any point. The distance BC is equal to the pitch circumference of the wheel. If these details were plotted for a lead of 6 in., it would be possible to measure the angle with a protractor, but such a method is not sufficiently accurate.



Consider a wheel in which the pitch diameter is to be 3 in. and the lead 6 in. This gives us enough information to calculate the tangent

Fig. 87.—The helix angle

of the angle, by dividing the pitch circumference— $3 \times 3.1416$ —by the lead—6 in.

$$\begin{aligned} \text{Tangent of angle BAC} &= \frac{CB}{CA} = \frac{3 \times 3.1416}{6} \\ &= 1.5708. \end{aligned}$$

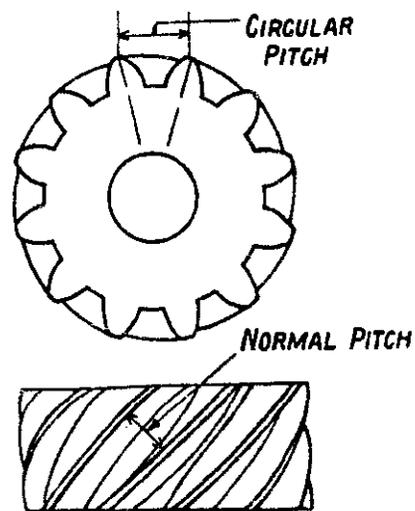
A table of tangents,

usually given with workshop handbooks shows that this represents an angle of  $57^\circ 31'$ , which is the angle at which the table will have to be set over to the helix.

Conversely, if the angle and the lead are given, then the pitch circumference is found by multiplying the tangent of the given angle by the lead, and the diameter by dividing this product by 3.1416.

In selecting a cutter for machining this type of wheel, another point presents itself—

Fig. 88.—Difference between circular and normal pitches  
the difference between the circular pitch and the normal pitch. This is illustrated in Fig. 88. Seen endwise, the wheel teeth have an apparent pitch that is unreal, though it is clearly the circular pitch. The



other view shows the same wheel in plan, from which can be seen the normal pitch, or width between successive teeth ; this is clearly the measurement needed to select the cutter required.

### **Grinding Gear Teeth**

The use of the grinding process for accurate gear-cutting is increasing, though generally speaking this may be said to be limited to precision work. The process can be used in two ways : either for finishing gears, cut in the milling machine, to a high degree of accuracy, or for cutting them entirely by grinding. In either case, the profile of the grinding wheel has to be very closely controlled by a diamond truing device, governed in its turn by accurate templates. Here, again, there are two systems at least : the first is the simplest possible, where the wheel face is given a shape that is true to the finished involute, and the second provides a true rack shape. In this case, the teeth are generated by a rolling motion of the head carrying the wheel blank, while the wheel is moved to and fro by a ram at the prescribed rate to give the correct cutting speed.

Gears that have to be specially hardened before use are ground to a high degree of finish after the hardening process, thus removing any traces of distortion. Though grinding produces a higher degree of finish than almost any other workshop process, it does not require a glass of high magnifying power to reveal the real nature of this finish. The use of burnishing to smooth out even the finest scoring produced by grinding is worth while. Here the cut gear is rotated between three hardened steel gears while pressure is applied. After only a few seconds, a very finely burnished surface is given to the soft gear, consisting of a skin of compressed metal that makes for an increased life. However, the finished tooth profile will have been checked and, if necessary, corrected. Burnishing is usually carried out on a milling machine. The two lower gears are supported in a fixture secured to the table ; the upper driving gear is fixed to the spindle, and the gear under treatment is held under pressure between the three.

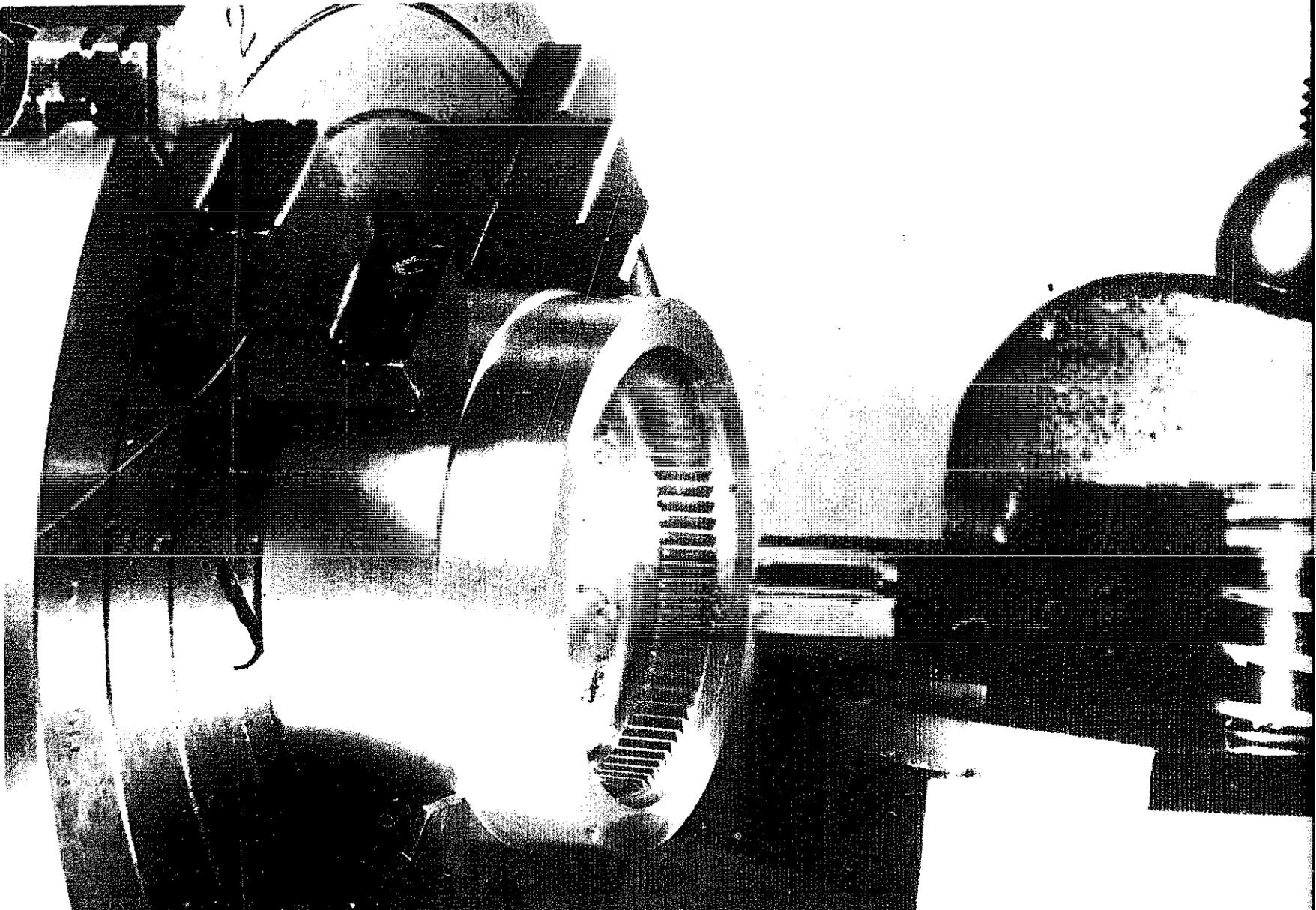


Fig. 80A—Cutting an internal ring gear

£11.25 net  
in UK only

ISBN 0 85242 532 5